

Damage Detection In Laboratory Concrete Beams

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Abstract

The aim of the investigation reported in this paper is to clarify to what extent damages in reinforced concrete can be detected by estimating changes in the vibrational properties. A series of damages were introduced by applying static load cycles of increasing magnitude to two concrete beams: a beam with a typical reinforcement ratio, and a beam with a small reinforcement ratio. The modal properties of the beams were found exciting the beams by a series of pulses and identifying the properties using ARMA and ARMAX models. It was found, that extremely small damages could be detected, that the significance of detection was only slightly improved using the measured input signal, and finally that it was easier to detect damage in a normally reinforced beam than in a lightly reinforced beam.

Nomenclature

n, m, p, i	: Integers
F	: Force
u	: Displacement
x_t	: Input time series
y_t	: Response time series
e_t	: Noise time series
Δt	: Sampling interval
Φ_i	: AR parameters
Θ_i	: MA parameters
λ	: Root in characteristic equation
ω_i	: Angular eigenfrequency
f_i	: Eigenfrequency
ζ_i	: Damping ratio
σ	: Standard deviation
d	: Damage state
S	: Significance indicator
U	: Unified significance indicator

Introduction

Reinforced concrete structures might experience many different kinds of damages: corrosion of the reinforcement, mechanical damages due to cracking and debonding, and deterioration of the concrete due to chemical actions from the environment.

Some of these damages are difficult to detect by traditional means like strength tests or visual inspection. In some cases cracks might be hidden by secondary structures, or possible damages might be inside the structure only visible though the change of the overall properties.

To assist in assessing the structural integrity of reinforced concrete structures vibrational based techniques might be useful. A comprehensive review of the basic ideas in vibrational based inspection of civil engineering structures might be found in Rytter [1]. Recently some attempts have been made in defining damage indication measures for reinforced concrete structures damaged by static or cyclic loadings, Garstka et al [6], Sadeghi [7], and cracking has been showed to significantly influence the modal properties of concrete beams, Almansa [8].

It seems not clear however, to what extent small damages typical for ordinary service loads might be detected by a simple measurement of the dynamical response. In order to assist in a better understanding of the practical possibilities using vibrational based techniques in the inspection of reinforced concrete structures, a series of measurements were carried out on two concrete beams, and the acceleration response were processed in order to show the sensitivity of the vibrational based detection technique to damage introduced by static loading.

For real structures it is usually not practical to excite the structure artificially. Thus, in many cases, the loading must be taken as the natural load, for instance wave loads on an offshore structure or traffic loads on a bridge. This means, that in many cases the loading will be unknown. Therefore in this investigation data were processed by ARMA models, since these does not require knowledge about the loading. To compare with the ideal situation, the data were also processed using a full input-output relationship (ARMAX models were used).

It is well known, that the crack pattern in concrete structures is highly dependent upon the amount of reinforcement. An increase in the reinforcement will tend to increase the number of cracks, but at the same time the crack opening will reduce. To illustrate the influence of this effect, two beams with different reinforcement ratios were tested, one beam with a typical reinforcement ratio (normally reinforced beam), and a beam with a quite low reinforcement ratio (lightly reinforced beam).

Test arrangements

The tested concrete beams had a 100×100 mm cross-section, a length of 1250 mm, and the concrete was a

dense high quality concrete with a compressive strength of 80 MPa, and mix recipe close to the one recently used in the Danish Great Belt Project.

The normally reinforced beam was reinforced with 4 ribbed bars, diameter 5 mm corresponding to a reinforcement ratio of 0.78 %, and the lightly reinforced beam was reinforced with 2 ribbed bars, diameter 4 mm corresponding to a reinforcement ratio of 0.25 %. The beams had no shear reinforcement, but they were designed in such a way, that the risk of shear failure was eliminated.

The beams were tested in three point bending with a span of 1200 mm, figure 1. The beams were loaded in displacement control up to a certain displacement and then reloaded. This procedure was repeated several times. Before the test started and after each load cycle, the dynamical response of the beam was measured in a separate test arrangement. The dynamical response was tested under free-free conditions as indicated in figure 1. The beams were excited by a series of pulses introduced at the lower end of the beam by a B&K impact hammer, and the response was recorded at the other end of the beam by a single accelerometer. The response signal was band-pass filtered between 80 and 1000 Hz (a Rockland 2382 filter), and the input and output signals were sampled at 3500 Hz using 16 bit simultaneous sampling (DT-2829 data-acquisition board). At each dynamical test 5 records of 10 seconds each were taken.

The crack pattern was different for the two beams. The normally reinforced beams developed many well distributed cracks with a small crack opening. The lightly reinforced beam developed only a few cracks close to the mid-section, and the cracks opened much more and on an earlier stage than for the normally reinforced beam. The crack patterns are indicated in figure 2.

The beams were loaded using a traditional servo-hydraulic loading system in displacement control. Using this loading system, the displacement u is increased with a constant speed. As long as the response of the material is linear, also the force F is increased with a constant speed. A drop in the force-speed indicates material softening, Elfgren [5]. The first reloading of the beams were taken at the point where a significant drop in force-speed could be detected. For the lightly reinforced beam, this point is close to a local maximum at the force-displacement curve.

During softening of the concrete micro cracks develop over a small region of the material. This fracture state might be modelled by the fictitious crack model, Hillerborg et al [9], Brincker et al [10]. At a later stage, the micro-cracks form into a real crack where no stresses are transferred, and a visible crack is developed. Using the fictitious crack model it has been shown, Ulfkjær et al [11], that the local maximum on the force-displacement curve of an un-reinforced beam correspond to a fracture state where no real crack is developed. Only a fictitious crack is present at this stage, and a real crack will first start to develop after the maximum is reached. Similar results have been obtained for reinforced beams, Ulfkjær et al [12]. This means that the first damage state (after the first loading and re-loading of the spec-

imens) should correspond to a state of a very small damage. After the first loading no visible crack was observed, and it is believed, that only small micro-cracks in the cement paste and micro-cracks between the paste and the aggregate particles were present at this stage.

Both beams were loaded and re-loaded 7 times. The load cycles are shown in the force-displacement diagrams in figure 2. Cracks visible for the naked eye were present in the normally reinforced beam after the first five, and in the lightly reinforced beam after the first three load-cycles.

Modal estimation

The modal parameter were extracted from the response time series using a so-called Auto Regressive Moving Average (ARMA) model, Ljung [2], Söderström et al [3]. These models have been developed mainly for application in economics and electrical engineering, but since they are considered to be a more effective way of estimating modal parameter than FFT-based techniques, Davies et al [13], their use on structural systems has been increasing during the recent years, Pandit et al [16], Safak [15].

Given a time series $y_t = y(t\Delta t)$, $t = 0, 1, 2, 3, \dots$ where Δt is the sample interval, an ARMA model of order (n, m) is defined as

$$y_t = \sum_{i=1}^n \Phi_i y_{t-i} - \sum_{i=1}^m \Theta_i e_{t-i} + e_t \quad (1)$$

Φ_i are the auto regressive (AR) parameters describing the response y_t as a linear regression on the past values, and Θ_i are the moving average (MA) parameters describing the response y_t as a linear regression on the past values of an unknown time series, e_t . Now, since the response y_t might be considered as a linear regression problem, the last term in eq. (1) might be considered as the term describing the deviation between the measured time series y_t and the response predicted by the ARMA-model. Thus, using minimum least squares, the best fit correspond to minimising the variance of the time series e_t .

It might be shown, that an ARMA model of order $(2n, 2n - 1)$ is the covariance identical discrete model of a continuous system with n degrees of freedom, Kozin et al [14].

For the tested beams only the two first eigenfrequencies were measured. Thus a ARMA (4,3) should be sufficient. However to describe the influence of filters and higher order modes, it was found, that to have a good fit it was necessary to use an (6,5) model. Using ARMA models, the correct model choice is essential, thus, after choosing the model, the model must be validated by different kinds of tests. The models all fitted well except for the last damage state, i.e. the damage state close to the ultimate deformation of the beams. Thus, for both of the beams, data for the last damage state

was excluded from the analysis.

A small segment of a typical time series of the load process is shown in figure 3a, the corresponding response process is shown in figure 3b. ARMA models were estimated for all the individual time series, 35 time series for each of the beams. Once the ARMA parameters are estimated, an analytical expression for the spectrum is available, Pandit and Wu [4]. In figure 3c the ARMA spectrum is compared to the corresponding FFT spectrum.

When the AR parameters are known, the modal parameters of the beam are found from the $2n$ roots λ of the characteristic polynomial, Pandit et al [16]

$$\lambda^{2n} - \Phi_1 \lambda^{2n-1} - \dots - \Phi_{2n-1} \lambda - \Phi_{2n} \quad (2)$$

The roots always appear in n complex conjugate pairs, one pair for each degree of freedom. The n 'th angular eigenfrequency ω and damping ratio ζ is found from the relation between the modal parameters and the n 'th complex conjugate pair of roots

$$\lambda = \exp((- \omega \zeta \pm i \omega \sqrt{1 - \zeta^2}) \Delta t) \quad (3)$$

In an ARMAX model an additional term is added so that the response y_t is a combination of autoregression and regression on a known load series x_t and on the unknown noise series e_t

$$y_t = \sum_{i=1}^n \Phi_i y_{t-i} + \sum_{i=1}^p B_i x_{t-i} - \sum_{i=1}^m \Theta_i e_{t-i} + e_t \quad (4)$$

Again the estimation problem is regression, and again the coefficients are found by minimising the variance of the noise time series e_t . The modal parameters still depend only on the autoregressive parameters Φ_i as given by eq. (2) and (3). Since more information is incorporated, usually the ARMAX model provides estimates with a smaller uncertainty than the ARMA model. ARMAX models were estimated only on the time series for the normally reinforced beam. A model of the order $(n, m, p) = (6, 5, 5)$ was used.

Damage indication

In damage detection, the first step is to clarify if any changes has taken place. The following steps including identifying the type, the size, and the location of the damage are often very difficult, especially in complex structures. Some results on identifying size and location in structures using neural networks might be found in Kirkegaard et al [17], [18].

The present investigation however, is limited to the first step of damage detection. The intention is only to investigate to what extend the damages introduced by the loading cycles shown in the force displacement diagrams in figure 2 might cause changes of the modal

parameters that are statistically significant.

As explained earlier, for each damage level five time series were taken. For each of the time series an ARMA (6,5) model was fitted and the eigenfrequencies and damping ratios corresponding to the two first modes were obtained. The virginal state modal parameters were for the normally reinforced beam $f_1 = 278.8$ Hz, $f_2 = 751.6$ Hz, $\zeta_1 = 0.40$ %, $\zeta_2 = 0.74$ %, and for the lightly reinforced beam $f_1 = 295.6$ Hz, $f_2 = 783.4$ Hz, $\zeta_1 = 0.34$ %, $\zeta_2 = 0.46$ %. From the five individual estimates of the modal parameters on each damage level the mean values and the standard deviations σ on the mean values were obtained using standard formulas for the empirical variance. The coefficients of variation were of the order of 0.02 % for the eigenfrequencies and 5 % for the damping ratios.

Let f_{nd} , ζ_{nd} denote the eigenfrequency and the damping ratio for mode n in damage state d . Damage state $d = 0$ correspond to the virginal state. Damage might then be indicated by plotting the relative drop in eigenfrequencies f_{nd}/f_{n0} as a function of the damage state d , figure 4a. Similarly the relative increase in the damping ζ_{nd}/ζ_{n0} might be plotted, figure 4b. From the graphs in figure 3a and 3b there seems to be a clear influence from the introduced damage. As expected, the eigenfrequencies drop, and the damping increase. It does not appear, however, to what extend the changes are statistically significant.

To indicate the statistical significance of the changes another parameter is useful. Consider the deviation $f_{n0} - f_{nd}$. Now, if f_{n0} and f_{nd} are assumed to be stochastically independent variables with standard deviations σ_{fn0} and σ_{fnd} , respectively, then the variance on the difference $f_{n0} - f_{nd}$ is $\sigma_{fn0}^2 + \sigma_{fnd}^2$. A useful measure for indication of statistically significant changes would be to take the ratio between the difference and its standard deviation, thus, the following significance indicator S is defined

$$S_{fn} = \frac{f_{n0} - f_{nd}}{\sqrt{\sigma_{fn0}^2 + \sigma_{fnd}^2}} \quad (5)$$

A similar significance indicator might be defined for the damping or for any quantity with known variance. For the damping the significance indicator is defined so that expected changes will increase the indicator

$$S_{\zeta n} = \frac{\zeta_{nd} - \zeta_{n0}}{\sqrt{\sigma_{\zeta nd}^2 + \sigma_{\zeta n0}^2}} \quad (6)$$

The defined significance indicators for the eigenfrequency are shown in figure 4c and for the damping ratio in figure 4d.

As it appears from the graphs, the change of the eigen-

frequency is highly significant even for the very first damage level. Since deviations larger than 2-3 must be considered significant, deviations of the order of 100 on the first damage level for the eigenfrequencies indicate that much smaller damages might be safely detected. Bearing in mind the small amount of damage introduced by the first loading cycle, this might seem surprising. However, it seems promising for the practical application of vibrational based inspection techniques in civil engineering.

For the damping ratios the indication is not as significant. For the first mode the change is of the order of 20 for the first damage state clearly indicating a significant change. For the second mode however, the changes are smaller. Generally, the significance indicators based on the damping ratios gives a weaker indication than the corresponding indicators for the eigenfrequency. This is due to a larger variance on estimated damping ratios.

Having obtained significance indicators S_i for the individual modal parameters as defined above, a unified significance indicator might be defined simply by adding the individual significance indicators. Thus the following unified significance indicator U might be defined for the beams as

$$U = S_{f1} + S_{f2} + S_{\zeta1} + S_{\zeta2} \quad (7)$$

The unified significance indicator for the two tested beams are shown in figure 5a. Since the normal reinforced beam develop many cracks and the lightly reinforced beam develop only a few cracks, it should be expected as it appears from the graphs in figure 5a, that the changes with damage state is more pronounced for the normally reinforced than for the lightly reinforced beam. Although the level of significance is lower for the lightly reinforced beam, small damages can be detected with a high significance.

As it appears from the comparison between significance indicator based on eigenfrequencies and damping ratios, the value of a significance indicator is highly dependent upon the variance on the estimated modal parameter. Thus, since variance is dependent on the estimation technique, care should be taken when choosing the technique for estimating modal parameters used in damage detection. To illustrate this phenomenon, and to investigate the loss of quality in the damage detection when discarding the information in the measured input signal, the unified significance indicator were estimated based on ARMA model (no input signal is used) and ARMAX models (the input signal is used). The results are shown in figure 5b. As expected, the ARMAX model estimates the modal parameters with a lower uncertainty than the ARMA models resulting in a higher level of significance in the damage detection. However, the unified significance indicator based on ARMAX estimation is only slightly better than the damage indicator based on ARMA estimation, figure 5b. Thus it might be concluded that knowing the input signal is not essential.

Conclusions

A normally reinforced and a lightly reinforced concrete beam has been submitted to increasing static load cycles in order to introduce damage of different severity. The first damage level correspond to a very small mechanical damage characterised by no visible cracks. The last damage level correspond to large plastic deformations of the order of the total deformation capacity of the beams.

The dynamic properties were measured under free-free conditions exciting the beams at one end with a series of pulses while measuring the response at the other end using only one accelerometer. Eigenfrequencies and damping ratios were estimated using ARMA models. A unified significance indicator describing the significance of the structural change has been defined. The indicator expresses the sum of all modal deviations from the virginal state divided by the standard deviation. This unified significance indicator was very sensitive to all damages introduced. Even the first damage state corresponding to a very small mechanical damage gave a clear indication of significant structural change. The results indicate, that it should be possible to detect damages that are smaller than the damages introduced in this investigation.

The two beams showed different crack patterns. The lightly reinforced beam developed only few cracks close to the mid-section, while the normally reinforced developed many cracks, well distributed over the most of the beam. As expected, the damage in the lightly reinforced beam gave a smaller indication of change than for the normally reinforced beam

Since the unified significance indicator involves the standard deviation on the estimated modal parameters, the quality of the damage indicator depends on the system identification technique. In order to illustrate the significance of using the information in the input signal, all data for the normal reinforced beam were also analysed by ARMAX models. As expected, the ARMAX based significance indicator was more sensitive than the *significance indicator based only on the ARMA model*. However, the difference was quite small. Thus, it can be concluded, that knowing the input signal is not essential for detection of damage in the actual beams.

Acknowledgements

This work was carried out under a grant from the Danish Technical Research Council. This support as well as support from the Structural Laboratory at the Department of Building Technology and Structural Engineering at Aalborg University is gratefully acknowledged.

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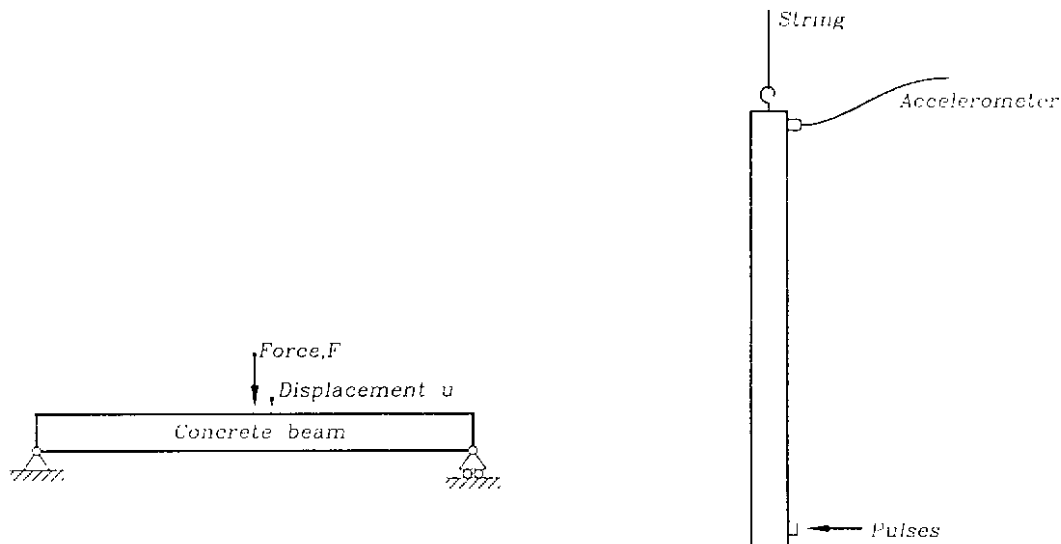


Figure 1. Test arrangements. Left: Loading of the beams in three point bending. Right: dynamic excitation under free-free conditions.

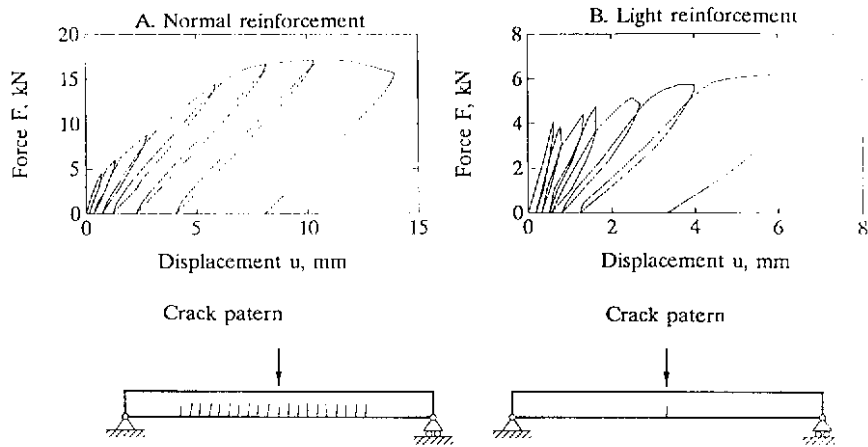


Figure 2. Test results for loading of the beams. Force-displacement curves and crack patterns for the two tested beams. A: normally reinforced, B: lightly reinforced.

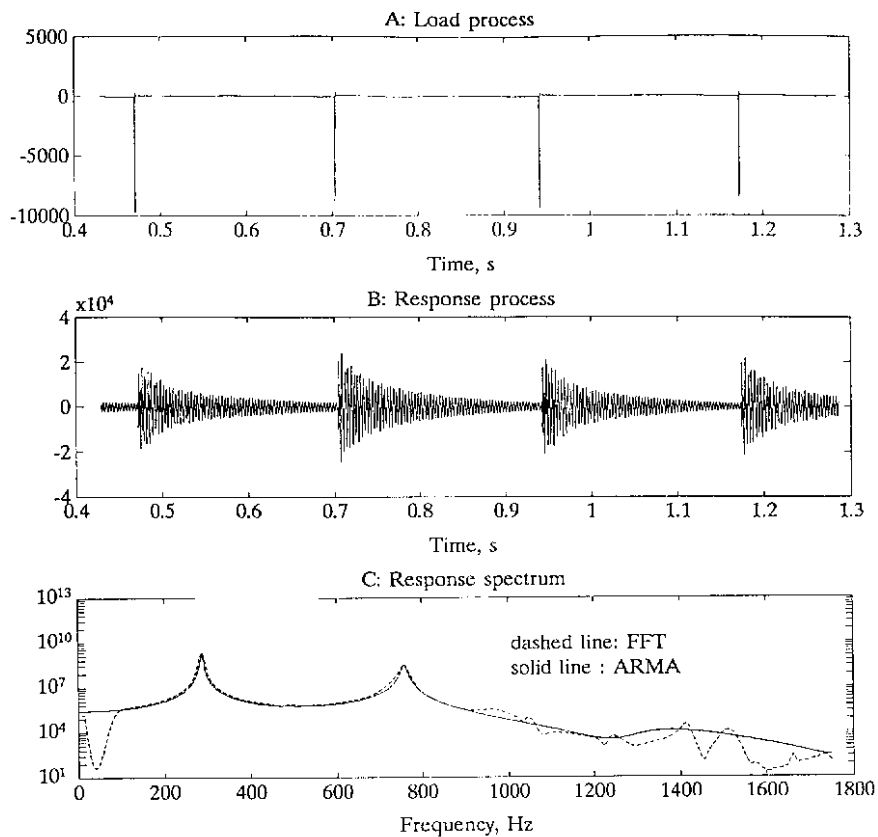


Figure 3. Typical records from dynamical test. A: the beams excited at one end by a series of pulses, B: response measured at the other end of the beam, C: response spectrum estimated by ARMA and FFT.

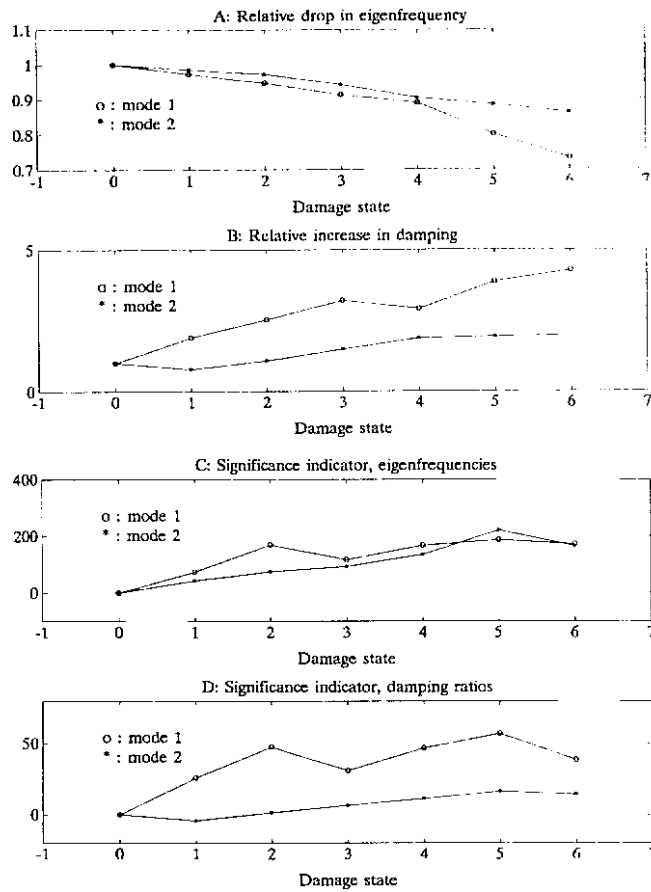


Figure 4. Damage indicators for normally reinforced beam plotted as function of the damage state. A: relative drop in eigenfrequency, B: relative increase in damping, C: the defined significance indicators for the eigenfrequencies, D: the defined significance indicators for the damping ratios.

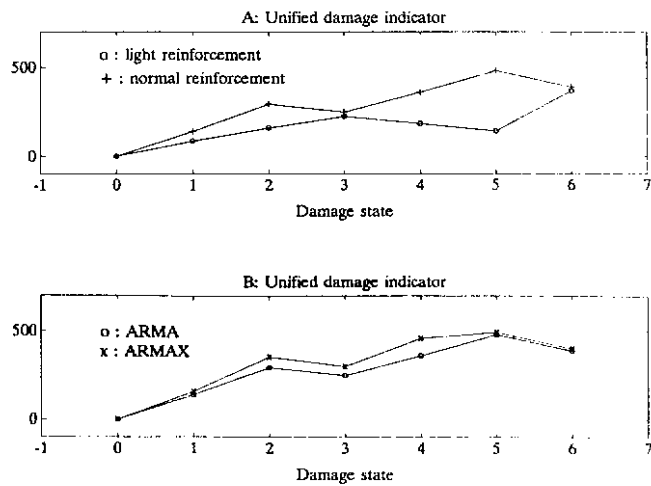


Figure 5. Unified damage indicator. A: unified significance indicator for normally reinforced beam estimated by ARMA and ARMAX models, B: unified significance indicator for the two beams estimated by ARMA models.