MODAL PARAMETER IDENTIFICATION FROM RESPONSES OF GENERAL UNKNOWN RANDOM INPUTS

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ABSTRACT. Modal parameter identification from ambient responses due to a general unknown random inputs is investigated. Existing identification techniques which are based on the assumptions of white noise and or stationary random inputs are utilized even though the inputs conditions are not satisfied. This is accomplished via adding, in cascade, a force conversion system to the structure's system under consideration. The input to the force conversion system is white noise and the output of which is the actual force(s) applied to the structure. The white noise input(s) and the structure's responses are then used to identify the combined system. Identification results are then sorted as either structural parameters or input force(s) characteristics.

NOMENCLATURE

Roman

c	damping coefficient
f	force
H	transfer matrix or function
i	index or $\sqrt{-1}$
j	index
k	stiffness
m	mass
n	white noise
s	Laplace variable
t	time
x	response

Abbreviations

ARMA	Auto Regressive Moving Average
ARV	Vector Auto Regressive (model)
ITD	Ibrahim Time Domain
RDD	Random Decrement

Subscripts

c	combined system
f	force system
s	structure

Superscripts

T	transpose

Greek and Symbols

ζ	damping factor
ϕ	mode shape
ω	circular frequency
$<$	angle

1. INTRODUCTION

Modal parameter identification from ambient responses has gained considerable attention in recent years[1,2]. The literature reveals a multitude of cases of vibration testing of bridges, buildings, off shore structures, aircrafts, spacecrafts, ground vehicles, among others, utilizing responses due to wind, waves, traffic, road roughness, propulsion systems . . . etc. The advantages of such techniques are quite evident: The normal operation of the structure under test is not interrupted; no excitation cost; no measurements of inputs; continuous if not unlimited response records; suitable for structural integrity monitoring.

However, identification from ambient responses possesses two main disadvantages: first the input energy may be low to excite the modes of interest; secondly the input is assumed to be white noise or stationary random.

The general identification theories as applied to modal parameters estimation of vibrating structures can be classified into different main categories depending on the nature of the loading. Usually the loads are assumed to fall into one of the three following categories:

- Known and measurable force inputs time histories and locations,
- no force inputs, (utilizing structure's free response due to *initial* excitation), and
- white noise inputs.

However, there exist many structural applications, as pointed out earlier, where it is either impractical or uneconomical to use, or satisfy the conditions of, the above mentioned inputs. At the same time, these types of structures or applications offer the readily available and economical ambient or operational responses.

Such applications have traditionally been analyzed implementing identification techniques such as Frequency Response Functions, ARMA models and Random Decrement Techniques; among others. Such approaches, however, are based on the classical assumption of white noise inputs; a condition that is not usually satisfied.

In this paper, these techniques which require white noise inputs will be extended to apply to cases of general force inputs. This is accomplished by adding a pseudo second order system, in series with the second order system representing the structure, to which pseudo white noise inputs are applied. The responses of the combined system loaded by white noise are in reality the actual structure's response to the general force inputs.

Simulated and experimental results are presented in support of the proposed approach. The techniques implemented here are applied to a full scale structure in another publication[3].

2. THEORY

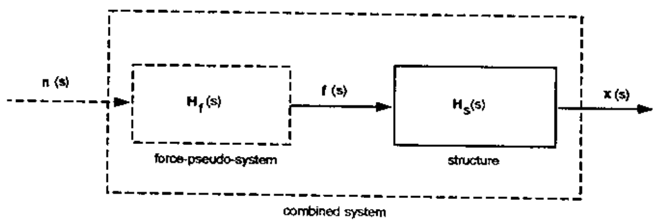

Figure 1. Block Diagram of Combined Structure's and Assumed Force's Systems

As shown in Figure 1, the structure whose transfer function is $H_s(s)$ has an input of $f(s)$ and an output of $x(s)$. The input $f(s)$ is not white noise. A pseudo second order system of a transfer function $H_f(s)$ is added in series to the system and is assumed to have the dynamic characteristics such that if the input to it is white noise, the output is the actual force to the structure $f(s)$. Now for the combined system in cascade the input is white noise and the output is the actual structure's response. Even though systems in cascade have been fully analyzed in dynamical systems theory[4], the objective of the ensuing proof is to address the identification aspect of systems in cascade, particularly vibrating systems, to ensure that the modal parameters of the structural system are independently preserved and can be uniquely identified.

The following equations relate the input to output for both the structural system and the force pseudo system

$$x(s) = H_s(s) f(s) \quad (1)$$

$$f(s) = H_f(s) n(s) \quad (2)$$

Thus the transfer function of the combined system becomes

$$H_c(s) = H_f(s) H_s(s) \quad (3)$$

Now let

$$H_s(s) = \sum_{i=1}^{2n} \frac{a_i}{s - \lambda_i} \quad (4)$$

and

$$H_f(s) = \sum_{j=1}^{2m} \frac{b_j}{s - \alpha_j} \quad (5)$$

Then the combined system transfer function becomes

$$H_c(s) = \sum_i \frac{a_i}{s - \lambda_i} \sum_j \frac{b_j}{s - \alpha_i} \quad (6)$$

$$= \sum_i \sum_j \frac{a_i b_j}{(s - \lambda_i)(s - \alpha_j)} \quad (7)$$

utilizing partial functions, equation (7) becomes

$$H_c(s) = \sum_i \sum_j \left\{ \frac{a_i b_i (\lambda_i - \alpha_j)}{(s - \lambda_i)} + \frac{a_i b_i (\alpha_j - \lambda_i)}{s - \alpha_j} \right\}$$
$$= \sum_i \frac{a_i}{s - \lambda_i}(A\lambda_i - B) + \sum_j \frac{b_j (C\alpha_j - D)}{s - \alpha_j} \quad (8)$$

where

$$A = \sum_j b_j \quad (9)$$

$$B = \sum_j b_j \alpha_j \quad (10)$$

$$C = \sum_i a_i \quad (11)$$

$$D = \sum_i \alpha_i \lambda_i \quad (12)$$

Equation (8) verifies the stipulation that the modal parameters of the structural system and the force pseudo system are preserved and separable. The poles, in the denominator, are unaffected by combining the two systems. Thus frequencies and damping factors identification is expected to be correct. As well, the residues in the partial fractions are simply multiplied by a constant for each mode. Thus the mode shapes remain uniquely identifiable.

3. THEORY VERIFICATION AND TESTING

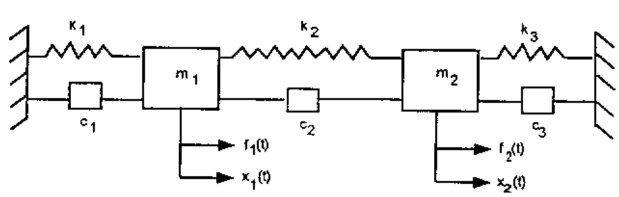

Figure 2. System for Simulation

To test the above theorems, a simulated test of a two degrees of freedom system is performed. The system is shown in Figure 2. Real experimental results are reported in another publication[3]. For the simulated experiment a white noise process is inputted to the force pseudo system which in this case assumed to be a single degree of freedom second order system the output of which is

$$F(w) = \frac{1}{w_f^2 - w^2 + 2i\zeta_f w_f w} \qquad (13)$$

The load to the "structure" under test is simulated using a covariance equivalent ARMA model:

$$F(t_i) = AR_1 F(t_{i-1}) + AR_2 F(t_{i-2}) + N(t_i) + MA_1 N(t_{i-1}) \qquad (14)$$

where $N(t)$ is the white noise and AR_1, AR_2, MA_1 are functions of w_f, ζ_f and sampling rate ΔT. $F(t_i)$ is applied equally to both degrees of freedom; thus $F_1 = F_2 = F$. Figure 3 shows an example of the spectrum, time history and normal probability plots for the white noise and the output of the pseudo force system which is the input to the structural system. The force pseudo system for these plots has w_f = 15.2578 and ζ_f = 0.005. These numbers are merely for illustration. For structural simulation w_f is taken as the average of the two natural frequencies of the system

The structural system's parameters were chosen as:

$$k_1 = k_3 = 150,\ k_2 = 20 \quad m_1 = 1.0 \quad m_2 = 2.0 \qquad (15)$$

and the damping matrix was selected as nonproportional of the form

$$C = 0.02M + 0.001K + \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \qquad (16)$$

Thus the system modal parameters were $w_1 = 9.0947$ r/s $w_2 = 13.1256$ r/s and $w_f = 11.1101$ r/s (average of ω_1 and ω_2). For damping factors $\zeta_1 = 0.0097$, $\zeta_2 = 0.0102$ and $\zeta_f = 0.005$.

The theoretical mode shapes we calculated to be:

$$\begin{array}{ll} \phi^1 = [1 \quad 4.3597]^T & <\phi^1 = [0 \quad 2.7613]^T \\ \phi^2 = [1 \quad 0.1148]^T & <\phi^2 = [0 \quad 176.0168]^T \end{array} \qquad (17)$$

Figures 4 shows spectra and time histories for the outputs.

The random decrement technique[5,6] "RDD" was used to convert systems random responses into free decay responses or correlation functions. Figure 5 shows the auto and cross RDD signatures using mass 1 as the triggering measurement and triggering was at every positive point. Figure 6 shows RDD signatures for triggering at local extremum of mass 1 response.

Modal parameter identification was performed using ARV[7,8] and ITD[9] methods. Table 1 shows identification results using signatures of Figure 5, and Table 2 is for those signatures of Figure 6.

From identification results, it can be seen that the system's characteristics as well as the force characteristics were identified.

Table 1. Modal Parameters estimated from RDD-Signatures (Every positive point trig. condition) by ARV and ITD Methods, and % Error from Theoretical Values

Parameter	Theory	ARV	% error	ITD	% error
ω_1	9.0947	9.1085	0.1517	9.1075	0.1407
ω_2	12.1256	13.1306	0.0881	13.1099	0.1106
$100.\zeta_1$	0.0097	0.0107	10.0810	0.0097	0.8711
$100.\zeta_1$	0.0102	0.0132	29.4118	0.0107	5.2745
Φ_1^1	1.0000	1.0000	—	1.0000	—
Φ_2^1	4.3597	4.3536	0.1899	4.3609	0.0292
$< \Phi_1^1$	0.0000	0.0000	—	0.0000	—
$< \Phi_2^1$	2.7613	7.3072	164.60	6.4866	134.9111
Φ_1^2	1.0000	1.0000	—	1.0000	—
Φ_2^2	0.1148	0.1170	1.9164	0.1199	4.4774
$< \Phi_1^2$	0.0000	0.0000	—	0.0000	—
$< \Phi_1^2$	176.0168	62.2515	7.8204	165.7310	5.8436
ω_f	11.1101	11.0994	0.0963	11.1332	0.2079
ζ_f	0.0050	0.0053	6.0000	0.0064	28.0000
Φ_{1f}	—	1.0000	—	1.0000	—
Φ_{2f}	—	1.1787	—	1.1347	—
$< \Phi_{1f}$	—	0.0000	—	0.0000	—
$< \Phi_{2f}$	—	176.3223	—	175.4883	—

Table 2. Modal Parameters Estimated from RDD-Signatures (Local extremum trig. condition) by ARV and ITD Methods, and % Error from Theoretical Values

Parameter	Theory	ARV	% error	ITD	% error
ω_1	9.0947	9.0997	0.0550	9.1011	0.0706
ω_2	13.1256	13.1307	0.0389	13.10448	0.1585
ζ_1	0.0097	0.0083	14.9869	0.0082	15.8557
ζ_1	0.0102	0.0140	37.3725	0.0136	33.7445
Φ_1^1	1.0000	1.0000	—	1.0000	—
Φ_2^1	4.3597	4.3118	0.3418	4.3518	0.1929
$< \Phi_1^1$	0.0000	0.0000	—	0.0000	—
$< \Phi_2^1$	2.7613	5.6025	102.8930	5.5074	99.4495
Φ_1^2	1.0000	1.0000	—	1.0000	—
Φ_2^2	0.1148	0.1183	3.0488	0.1229	7.0557
$< \Phi_1^2$	0.0000	0.0000	—	0.0000	—
$< \Phi_1^2$	176.0168	160.9737	8.5464	162.0385	7.9415
ω_f	11.1101	11.1078	0.0207	11.1099	0.0018
ζ_f	0.0050	0.0063	25.4600	0.6569	31.3800
Φ_{1f}	—	1.0000	—	1.0000	—
Φ_{2f}	—	1.1.694	—	1.1659	—
$< \Phi_{1f}$	—	0.0000	—	0.0000	—
$< \Phi_{2f}$	—	175.5213	—	175.1389	—

4. CONCLUSIONS

Ambient random responses treated with random decrement and time domain identification techniques are effective and economical in modal identification of structures. This approach, among others, is classically based on the assumption of stationary random input. Non-stationary random inputs results in the identification of extraneous modal parameters which belong to the forcing system rather than the structure being tested. However, it is shown that forcing function dynamic characteristics have no effect on the accuracy of structural parameter identification. Techniques need to be developed to assist in sorting out structural dynamic properties from those of the inputs.

5. ACKNOWLEDGEMENTS

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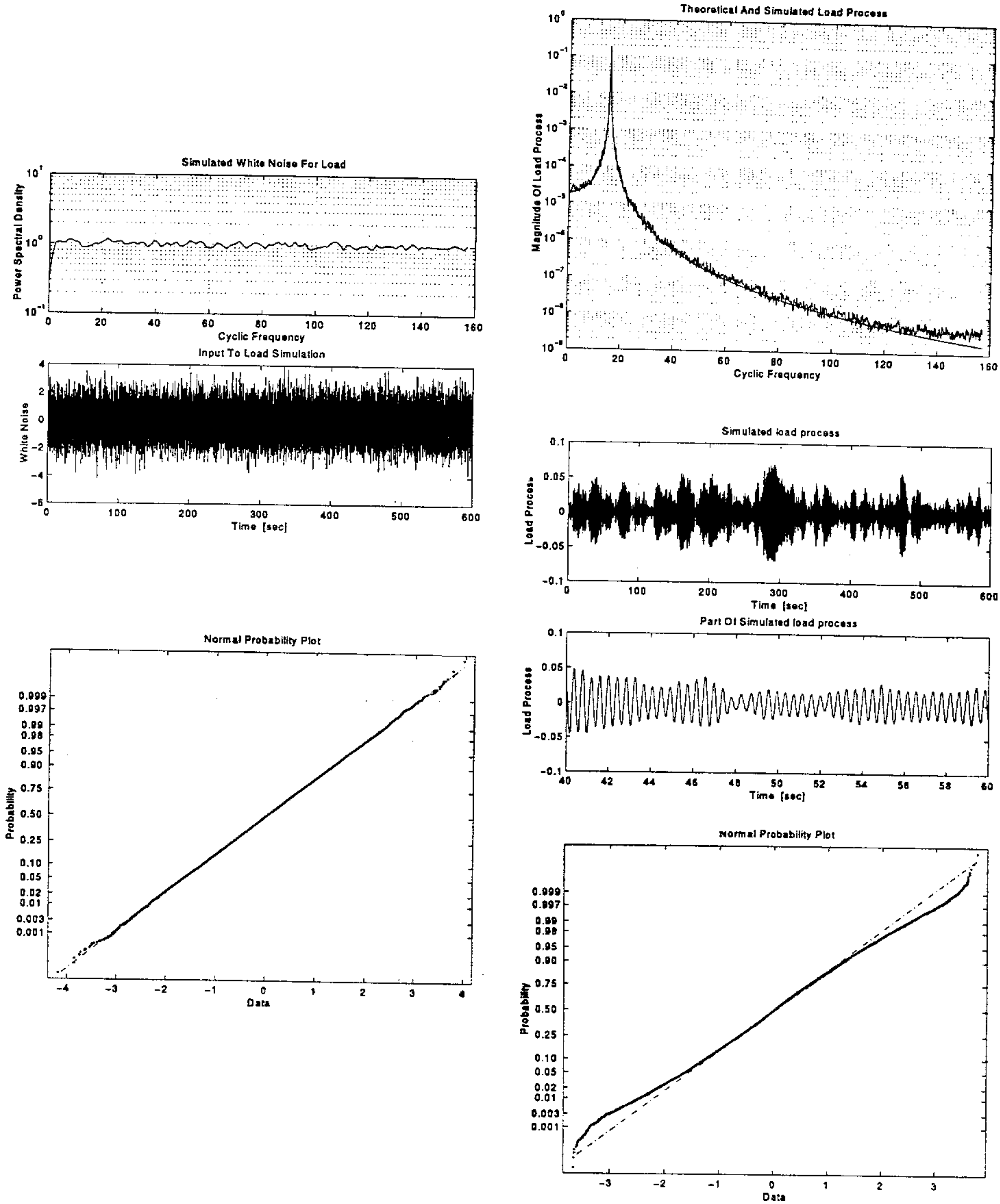

Figure 3. Spectra, Time Histories and Distribution of White Noise and Input to Structure

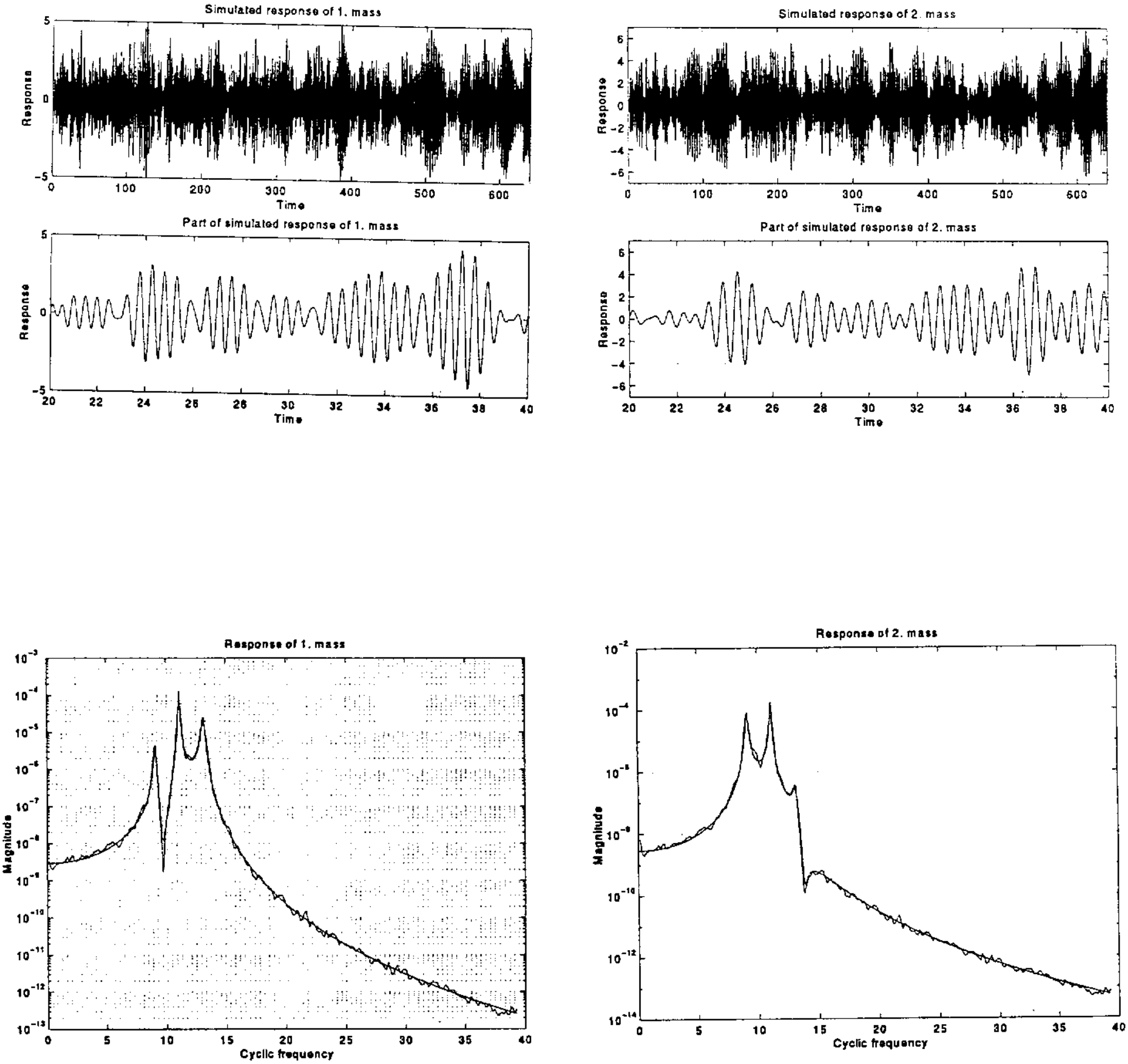

Figure 4. Time Histories and Spectra of Responses

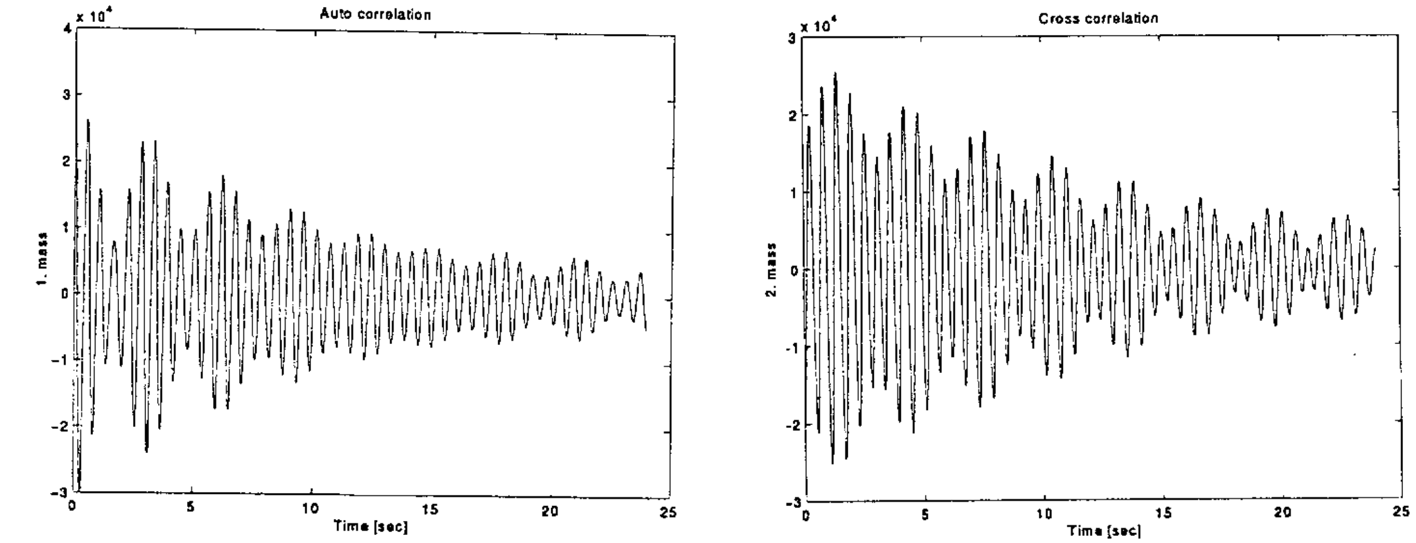

Figure 5. Random Decrement Signatures with Triggering on Every Positive Point of Mass 1

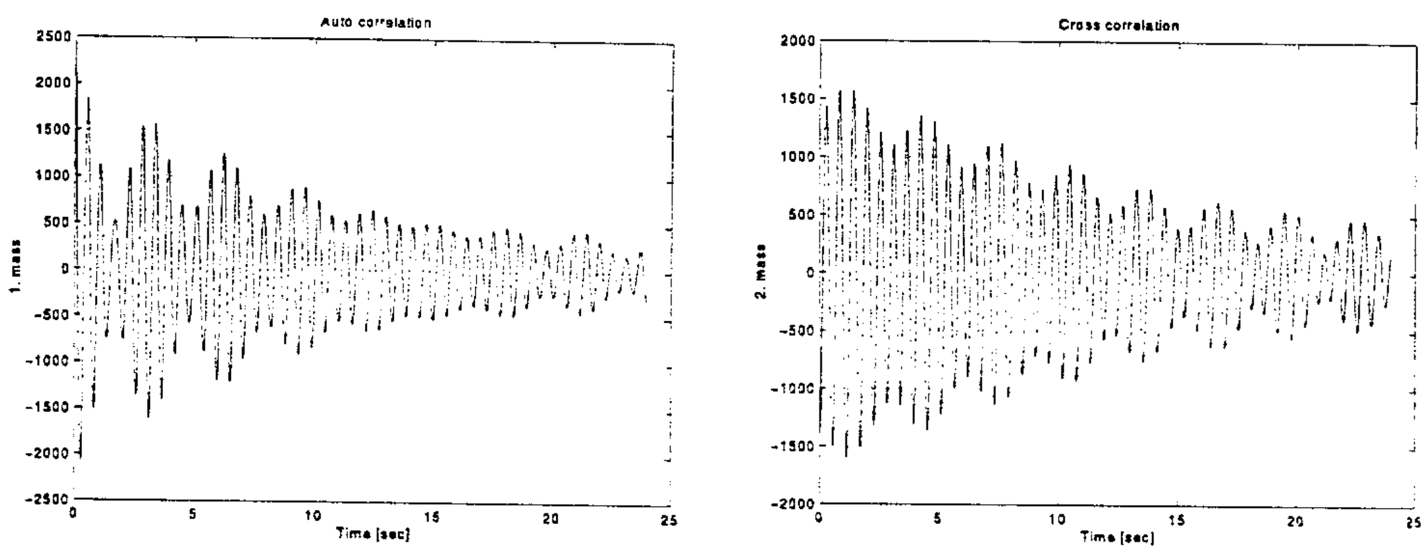

Figure 6. Random Drecement Signatures with Trigerring on Local Extremum of Mass 1