# AMBIENT DATA TO ANALYSE THE DYNAMIC BEHAVIOUR OF BRIDGES: A FIRST COMPARISON BETWEEN DIFFERENT TECHNIQUES

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***ABSTRACT.*** *The dynamic analysis of Queensborough bridge data recorded by EDI Ltd., based in Vancouver (Canada), under environmental excitation, is a chance to involve a few different research groups already working in this field.*
*Three different research units have employed alternatives methodologies to investigate and analyse the bridge data. With a complete agreement, they decided to compare the results of these analysis looking for a reliable common methodology. The aim of the present paper is a sort of a resume of different techniques, together with their advantages and drawbacks, and a comparison of the results obtained via these techniques in order to select the most reliable aspects and to merge them for creating a common strategy of approaching the bridge monitoring matter. All this, seeking to define an optimum standard methodology to be proposed to those institutions and maintenance companies daily operating in this field.*

## NOMENCLATURE

| | |
|---|---|
| $C_y$ | :Trig condition on $y$ |
| $D$ | :Random decrement signature |
| $D(t,f)$ | :Cohen distribution |
| $f$ | :Frequency |
| $g(\theta,\tau)$ | :Cohen transform kernel function |
| $N$ | :Number of trig points |
| $R_{xy}$ | :Cross correlation between $x$ and $y$ |
| $R'_{xy}$ | :Time derivative of $R_{xy}$ |
| $t_i$ | :Discrete time point (trig point) |
| $t$ | :Time |
| $t'$ | :Time dummy variable |
| $u[i,k]$ | :residual at the $i^{th}$ DOF |
| $x[i,k]$ | :sample acquired at the $i^{th}$ DOF |
| $x$ | :Response time series |
| $x^*$ | :complex conjugate of $x$ |
| $y$ | :Response time series |
| $\theta$ | :Frequency dummy interval |
| $\sigma$ | :shape parameter of kernel $g(\theta,\tau)$ |
| $\tau$ | :Time segment |
| $\zeta$ | :Damping ratio |

## 1. INTRODUCTION

The three proposed methods have different process layouts. The Research Unit of Politecnico di Torino, Department of Mechanics (T-MEC), proposes an intrinsecally one-stage procedure in the time domain, with a previous pre-processing by band-pass filtering. The time-series of input data are elaborated by a multidimensional regression leading to the *identification of modal parameters*.
The Units of Aalborg University and Politecnico di Torino, Dept. of Structural Engineering (T-STRUC), propose two different two-stage procedures. The first stage, in both cases, is a transformation algorythm: a time domain transform for the first case, a time-frequency distribution for the second one. The second stage is the application of a regression technique to detect the structural response, of parametric (Aalborg) and non-parametric type (T-STRUC).

## 2. COMBINING THE RDD AND THE ITD TECHNIQUE

The idea of this approach is to combine the Random Decrement (RDD) and the Ibrahim Time Domain (ITD) Technique. This paragraph gives a short description of this approach applied on the bridge data, details are reported in [3]. The Random decrement technique is a simple way of estimating short sequences of data representing the physical properties of a system excited by random loads. A short sequence of data produced by this technique is called a RDD signature. Now, let $y(t)$, be the time series of the responses measured on channels. For the time series $y(t)$ an estimate of the RDD signature is obtained by simple averaging of the form:

$$\hat{D}_{ij}(\tau) = \frac{1}{N}\sum_{k=1}^{N} y_i(t_k + \tau)\Big|C_j$$

where $C_j$ is some kind of trig condition applied to the time series $y_i(t)$, $t_k$ are the trig points, i.e. the times at which the trig condition is satisfied and $N$ is the number of trig points.

It might be shown that, for any trig condition, the RDD signature is an unbiased estimate of a combination of the cross covariance function $R_{ij}(\tau)$ and its derivative $R'_{ij}(\tau)$ [1]. Thus, for white noise excitation, the RDD signature is simply a free decay. The initial conditions might not be known, but a set of initial conditions does exist giving a free response of which the RDD signature is an unbiased estimate. Thus, for this case, the RDD signature represents the true physics of the system.

In the case of general random loading, it is possible to show that the RDD signature might be considered as the free response of the true system interacting with a pseudo-physical system describing the loading [2]. The non-structural modes corresponding to the degrees of freedom in the loading system does not change the physical parameters of the structural system. Thus, the structural system might be identified from the RDD signatures even though the loading is not white noise.

The structural parameters are extracted using the Ibrahim Time Domain technique [4]. Using this technique, the RDD estimates are ordered in the discrete time response matrices $\mathbf{x}_i(k)$ with the elements $x_{irs}(k) = \hat{D}_{ir}((k+s)\Delta t)$ where $\Delta t$ is the sampling time. Since all free decay responses are linear combination of the modes it is possible to show that

$$\mathbf{x}_i(k) = \mathbf{A}\mathbf{x}_i(k-1)$$

This equation is solved as an over determined system of linear equations for determination of the square matrix $\mathbf{A}$ using the least squares approach. The eigenvalues and the eigenvectors of $\mathbf{A}$ provide the poles and mode shapes of the system. Thus, for every full set of RDD signatures, $M$ estimates of the mode shapes and poles are obtained. These results are averaged. The number of degrees of freedom in the model is controlled by the size of $\mathbf{A}$.

Since the mode shapes, eigenfrequencies and damping ratios are found by least square fitting on unbiased free response estimates, damping ratios and eigenfrequencies estimated by this technique are expected to be free of serious systematic errors, and thus, the variance on the parameters might be used as uncertainty measure.

Advantages

- Speed. Since the RDD estimates are obtained by averaging, and since the ITD technique only requires to solve a set of linear equations and to solve an eigenvalue problem, the calculation time is close to the smallest possible.
- Accuracy. Since no bias is introduced in the estimation process, and since cross information is used, the accuracy of the modal analysis must be reasonable. However, since the modal analysis is based on fitting the RDD estimates, and since the RDD estimates cannot contain all information hidden in the original time series, the technique will not be as accurate as a technique based on fitting a model to the raw time series (like ARMA models).
- Amplitude dependency. The RDD estimates might be obtained for a set of trig conditions corresponding to different amplitude levels. Thus, by estimating modal parameters for such a set of RDD signatures, it is possible to investigate amplitude dependency of the modal parameters due to mass loading or non-linearities

Disadvantages

- Noise modes. Since the noise is not directly modelled in the ITD technique, and since noise will be present in the RDD signatures, a large number of noise modes has to be estimated to obtain a reasonably good fit. The noise modes must be identified and discarded by estimating a large number of different models for a given set of RDD signatures using stabilisation diagrams and judging the sensitivity of modal estimates.

## 3. SHARP BAND-PASS SELECTIVE FILTERING OF EXPERIMENTAL DATA IN THE TIME-FREQUENCY DOMAIN

Many investigations in the field of structural identification are employing time-frequency representations (TFR) of linear type (Short Time Fourier Transforms (STFT), Wavelets). The application of bilinear (or quadratic) transforms allows to eliminate the time and frequency resolution constraints posed by STFT's since, unlike the latter, they are not based on signal segmentation. In particular, Cohen class transforms make it possible to obtain quadratic time-frequency distributions which enjoy the properties of invariance relative to time and frequency signal translations. These properties are of essential importance to provide a correct physical interpretation of the phenomena being investigated [5,6]. It can be shown that all distributions which are invariant to time and frequency translations can be written in the form proposed by Cohen:

$$D(t,f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t'+\frac{\tau}{2})x^{*}(t'-\frac{\tau}{2})g(\theta,\tau)e^{-j2\pi\theta(t'-t)}e^{-j2\pi f\tau}d\theta dt' d\tau \tag{3}$$

where $x(t)$ is the signal to be transformed, $x^{*}(t)$ is its complex conjugate, $g(\theta,\tau)$ is the kernel of the transform. The indipendence of the kernel from time ($t$) and frequency ($f$) variables entails the invariance to translations which characterises Cohen class transforms.

*The bilinear structure leads to spurious terms* (interfering terms), due to cross-products between the different components producing the signal to be analysed, which are present in the final representation alongside the useful terms. In the latest literature, this shortcoming has been remedied by introducing transforms that are able to filter the interfering terms while useful terms are preserved. Choi and Williams [7] pointed out that effective filtering of interfering components can be obtained with an exponential type kernel, as defined by the following law, pertaining to the domain of the ambiguity function $(\theta,\tau)$:

$$g(\theta,\tau)=e^{-\frac{\theta^2\tau^2}{\sigma}} \tag{4}$$

The parameter $\sigma$ permits to choose kernel selectivity: for small values ($\sigma<1$), the $g(\theta,\tau)$ function is highly selective, whilst for higher values ($\sigma>1$) its filtering effect is lighter. Sequential energy variation leads to estimate an equivalent viscous damping as a time function. A constant time section can be likened to a Power Spectral Density providing instantaneous information on the power content of the different components. Modal response is identified by comparing the time histories obtained at several points through IIR (very narrow band) filtering at each frequency component [8]. Once filtered, the signals coming in from the different measuring points, all of them active simultaneously (single set-up), were aligned with high resolution techniques . After the alignment, phase data were extracted. The filtered signals relating to the different channels were then subjected to low-band filtering with 0.4 Hz cut off frequency, in order to extract the vibration envelopes of signals. The maximum amplitude of each envelope was subdivided into 20 equal intervals. The envelope curves were then subdivided into 20 segments corresponding to such intervals.

After that, the mean amplitude within each interval was correlated to the mean amplitude obtained over the same time interval by the envelope of the data recorded at reference point 9. This procedure results in plottings describing the relative amplitude of modal vibration as a function of the signal's energy level .

This relative amplitude was seen to be substantially constant, provided that the energy level exceeded a certain range of values, characterised by the dominance of the background noise. The value of relative amplitude, as determined over the significance threshold, when associated point by point to the phase data, which almost invariably stabilise around 0 or $\pi$, makes it possible to plot the modal shape diagrams.

The proposed approach to structural identification has some advantageous properties and some application problems:

Advantages:

- high resolution capacity in natural frequency and modal shape recognizing; e.g.,it is possible to observe the very small relative difference between east and west-edge modal shapes of the flexural modes. Slight frequency changes, due to heavy vehicle transits, are readable on time-frequency distributions;

- robustness; it is possible to achieve the correct shape even in a context of close modal superposition (third and fourth mode). Moreover, the wide scattering of phase and amplitude values is a symptom of a different closely near mode; besides, in Choi-Williams transforms, a close modal coupling can be recognized due to the different energy content ratio of coupled modes along the time axis;
- wide information supply; available informations are obtained about damping and signal to noise ratio. Even, non-linearity can be revealed (it's not our case) by a systematic deviation from constant value of the amplitude ratios due to the frequency peak-line sway with high energy levels.

Disadvantages

- The main application problems are connected to the non-immediately visible criteria for a fully automatic implementation of the method; besides the non-linear identification procedure is not yet defined and the identification of dampings is not, until now reliable.

## 4. THE ARMAV APPROACH

The application of Auto Regressive Moving Average Vector approach to the analysis of dynamic systems results in a time domain method that allows to compute the modal parameters of the structure [9].

The basic idea of this representation is that any output sample can be written as a linear combination of input and output values i.e.:

$$x[n] = \sum_{k=1}^{p} a[k]x[n-k] + u[n] + \sum_{k=1}^{q} b[k]u[n-k]$$

where x[n] is the generic output sample and u[n] the input.

The previous equation describes a (p,q) ARMA model and can be generalised into a vector form as follows:

$$\bar{x}[n] = \sum_{k=1}^{p} A[k]\bar{x}[n-k] + \bar{u}[n] + \sum_{k=1}^{q} B[k]\bar{u}[n-k]$$

with $\bar{x}[n]\,;\,\bar{u}[n] \in R^{s}$ and $A[k]\,;\,B[k] \in R^{s\cdot s}$.

It is possible to demonstrate that, for a linear and time invariant dynamic system, this general ARMAV(p,q) model can be reduced to a (2,1) model without any loss of generality. It is also possible to link the ARMAV poles with the natural frequencies and the mode shapes of the continuous time system thus simulated. It is worth saying that this last passage doesn't require any kind of curve fitting, neither in the time nor in the frequency domain, but only some trivial algebraic manipulations.

A great quality of this characterisation is the chance to deal with the output produced by a random input even if the input time histories are not but statistically known. Yet there is a limitation, regarding the stationarity of the input and output time series. If the input is non stationary its characteristics are not invariable and the ARMAV model can't properly fit the data since its parameters are constant.

Moreover it is important to notice that the system to be modelled via the ARMAV approach must be linear and time invariant and this is the assumption we forced on the Queensborough Bridge. This is not true strictly speaking because the bridge mass, depending on the traffic conditions, is surely changing and we don't know exactly the structure characteristics to state whether it is linear or not.

The ARMAV model is then to be considered correct as far as the system under test is linear, time invariant and excited by a stationary white noise. Any relaxation of these requirement will result in a partially incorrect analysis.

In the general formulation of the problem as stated above, it should be noted that the input has to be generated by uncorrelated random signals applied to all the system DOFs. If just some of the DOFs are excited the procedure could misbehave and this is the reason why, in our analysis, we used the modified algorithm presented in [9].

The practical application of the procedure to the actual data measured on the Queensborough Bridge proved quite satisfactory even if the requirement of stationary data is not satisfied. This confirms the robustness of the algorithm even if a wise usage is necessary to achieve reasonable results [10].

Advantages:

- the input has to be known just statistically
- the system is treated as a black box

- no need to perform any curve fitting
- robust algorithm

Disadvantages:
- non stationary output data have to be handled with care
- it is not possible to analyse non linear systems

## 5. COMPARATIVE EVALUATIONS

As it is observable from Fig. 1 and Table 1, the modal frequency estimates are almost equal for the three different approaches. The modal shape is reconstructed with a comparable reliability for uncoupled modes. The T-STRUC approach is somehow weeker for coupled modes. The different effectiveness in isolating coupled modes comes from the fact that the Aalborg and T-MEC methods use cross-correlated informations from different channels, more efficient to that aim than the auto-correlated ones adopted in T-STRUC model. On the other hand, the time-frequency distribution has a meaningful physical readability and can show slight natural frequency changes due to heavy running vehicles.
The modal damping is always critical to detect and the effective reliability is not known. The Aalborg approach uses full-time sample to calculate the data correlations. Slight non-stationarity, e.g. due to transient mass additions, could cause slow changes in correlation function amplitudes, similar to the effect of higher damping. Such effect, however, depends on the maximum time delay in correlation functions.
Also the T-MEC approach leads to the damping coefficient estimate through the complex nature of the eigenvalues. Damping could be more efficiently evaluated because the ARMAV algorithm is applied in short time windows, giving a better protection against non stationarity, but only if the the sampling ratio is well chosen and the signal-noise ratio is good enough.
The T-STRUCT model allows the evaluation of the damping factor through the free decay of filtered time histories after transient loading. This method, unlike the previous two, does not make any hypothesis about the stationarity of structural response, but a correct evaluation requires some experimental records in conditions of low traffic, to be sure that the decay after a transient event is free from disturbances.

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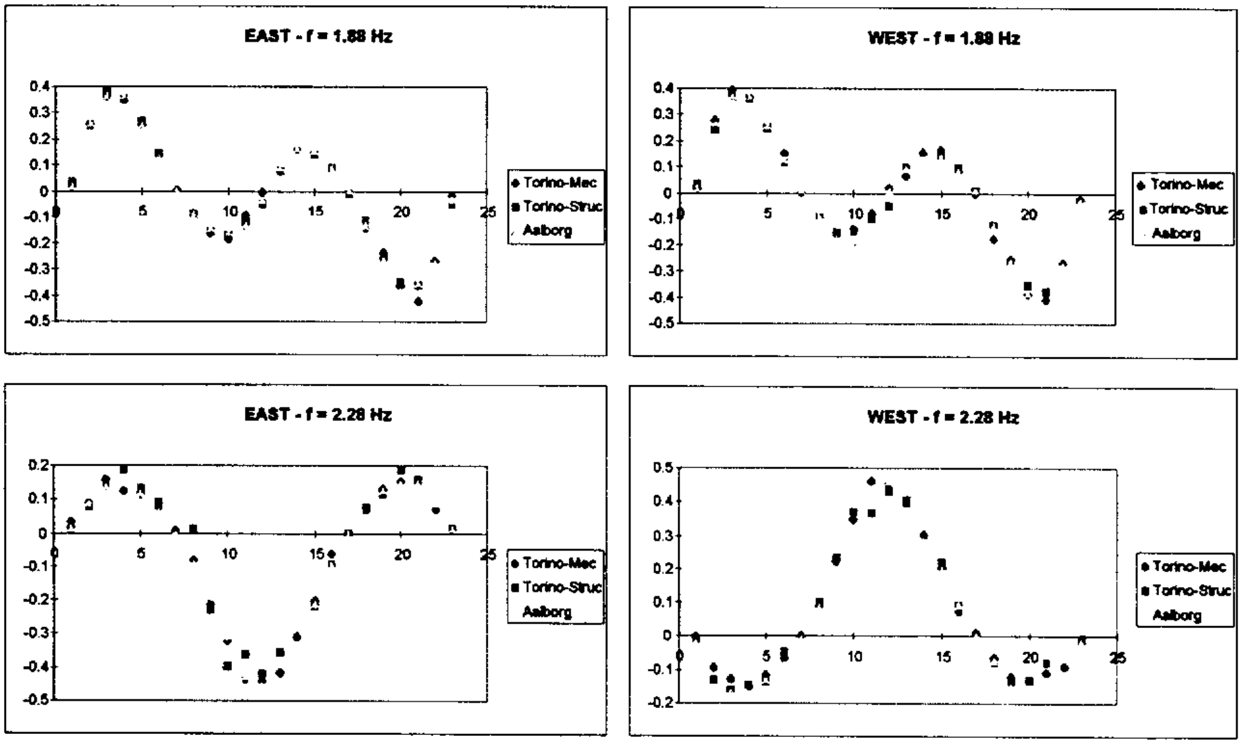


**Fig.1** Compared estimates of modal shapes

**Tab. 1** Compared estimates of modal parameters

| MODE | FREQUENCY [Hz] | | | FREQ. VARIANCE | | | DAMPING [%] | | | DAMP. VARIANCE | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | T-Mecc | T-Struc | Aalborg | T-Mecc | T-Struc | Aalborg | T-Mecc | T-Struc | Aalborg | T-Mecc | T-Struc | Aalborg |
| 1 F | 1.12 | 1.10 | 1.10 | | | 2.13E-2 | 1.94 | (5) | 7.36 | | | 4.32E-2 |
| 2 F | 1.88 | 1.87 | 1.88 | | | 1.61E-2 | 0.87 | 3.1 | 1.45 | | | 1.02E-2 |
| 3 T | 2.29 | 2.28 | 2.28 | | | 2.31E-2 | 0.49 | | 1.86 | | | 2.36E-2 |
| 4 F | 2.42 | 2.42 | 2.42 | | | 2.22E-2 | 0.84 | | 2.15 | | | 2.48E-2 |
| 5 T | 3.20 | 3.20 | 3.20 | | | 1.50E-2 | 0.78 | | 1.51 | | | 1.80E-2 |
| 6 T | 3.43 | 3.45 | 3.44 | | | 1.69E-2 | 0.58 | | 1.08 | | | 1.26E-2 |
| 7 F | 3.70 | 3.74 | 3.73 | | | 3.01E-2 | 1.4 | | 1.68 | | | 1.60E-2 |
| 8 T | 5.15 | 5.16 | 5.15 | | | 1.84E-2 | 0.25 | | 0.64 | | | 5.59E-2 |
| 9 F | 5.78 | 5.84 | 5.74 | | | 3.07E-2 | 0.96 | | 1.15 | | | 6.48E-2 |
| 10 T | 7.04 | 7.12 | 7.01 | | | 2.49E-2 | 0.65 | | 0.77 | | | 4.49E-2 |
| 11 T | 7.52 | 7.52 | 7.53 | | | 1.92E-2 | 0.82 | | 0.69 | | | 4.40E-2 |
| 12 T | 8.56 | 8.64 | | | | | 0.32 | | | | | |