VECTOR TRIGGERING RANDOM DECREMENT TECHNIQUE FOR HIGHER IDENTIFICATION ACCURACY

S. R. Ibrahim[1] J. C. Asmussen[2] R. Brincker[2]

[1]Department of Mechanical Engineering
Old Dominion University
Norfolk, Virginia, USA

[2]Department of Building Technology and Structural Engineering
Aalborg University, Sohngaardsholmsvej 57, 9000 Aalborg, Denmark

ABSTRACT. Using the Random Decrement (RDD) technique to obtain free response estimates and combining this with time domain modal identification methods to obtain the poles and the mode shapes is acknowledged as a fast and accurate way of analyzing measured responses of structures subjected to ambient loads. Using commonly accepted triggering conditions however, one is limited to use a combination of auto-RDD and cross-RDD functions with high noise contents on the cross-RDD functions. Using only the auto-RDD functions, estimated independently for each channel, causes the loss of phase information and thus the possibility of estimating mode shapes. In this paper, a new algorithm is suggested that is based on pure auto-triggering. Auto-RDD functions are estimated for all channel to obtain functions with a minimum of noise, but using a vector triggering condition that preserves phase information, and thus, allows for estimation of both poles and mode shapes. The proposed technique (VRDD) is compared with more commonly used triggering conditions by evaluating modal parameters estimated by time domain technique on simulated data.

NOMENCLATURE

Roman

a	Constant
b	Constant
A	State space matrix
f	Frequency or force
H	Transfer matrix
N	Number of averages
r	Random responses
t	Time
T	Time
x	Free response

Greek

α	Modal displacement
β	Modal displacement
ζ	Damping factor
λ	Characteristic root
τ	Time
ϕ	Modal displacement(s)

Symbols

[ ]	Rectangular matrix
{ }	Vector
| |	Magnitude
$\angle$	Phase angle

Abbreviations

ARMAV	Vector Auto-Regressive Moving Average
DOF	Degree(s) Of Freedom
ITD	Ibrahim Time Domain technique
RDD	Random Decrement technique
rms	Root mean square
VRDD	Vector Triggering Random Decrement technique

1. INTRODUCTION

Since its introduction by Henry Cole [1], Random Decrement, RDD, technique has been extensively investigated and used [2–7] as a powerful tool in Modal Identification. It economically and efficiently converts random responses, due to unknown or unmeasured stationary random input, to free-decay responses. Such responses are suitable for many modal identification techniques in both time and frequency domains. Such approach renders itself as an efficient and powerful tool to analyze ambient responses from all types of structures.

The basis of the RDD is quite simple. The forced responses of a structure can be written as:

$$\{r(t)\} = e^{[A(t-t_o)]}\{r(t_o)\} + \int_{t_o}^{t} [H(\tau)]\{f(t-\tau)\}d\tau \qquad (1)$$

The first part of equation (1) is the homogeneous solution of the system's governing differential equations of motion which is dependent on system's dynamic characteristics and not on the input. The second part is the particular integral and is dependent on systems' transfer functions and input.

In using RDD for modal identification, the phase relations between different measurements must remain unchanged or if changed the changes must be systematic and later corrections of identified mode shapes must be performed.

Thus, the RDD functions computed for the responses of equation (1) can be written as:

$$\{x(\tau)\} = \frac{1}{N}\sum_{i=1}^{N}\{r(t_i+\tau)\} \tag{2}$$

If $\{f(t)\}$ in equation (1) are stationary random and N is a large number of averages, the second part of equation (1) averages out. To ensure that the first part of equation (1), the free-decay responses, do not average out, a condition is associated with t_i. This condition is referred to as the triggering condition. Triggering condition is applied only to a selected measurement j. Among well known triggering conditions are:

$$r_j(t_i) = a \tag{3-a}$$
$$r_j(t_i) \geq a \tag{3-b}$$
$$a \leq r_j(t_i) \leq b \tag{3-c}$$
$$r_j(t_i) \geq 0 \tag{3-d}$$
$$dr_j/dt|_{t=t_i} > 0 \text{ and } r_j(t_i) = 0 \tag{3-e}$$

The reference measurement j can be any arbitrary measurement or the RDD computation can be repeated as many times as the number of measurements. changing the reference measurement every time, [5].

The response of measurement j, $x_j(\tau)$, is referred to as the auto RDD while for other measurements cross RDD results. This is analogous to auto-correlation and cross-correlation functions.

The auto RDD functions are known to have less noise than the cross RDD ones. This is due in part to the higher rms value of auto RDD functions.

The purpose of this work is to present a vector triggering approach which produces all auto RDD functions while maintaining phase relations between measurements. This is expected to reduce the noise to signal ratio in all resulting RDD functions; thus improve identification accuracy.

2. THEORY OF PROPOSED APPROACH (VRDD)

For the purpose of derivation, and without the loss of generality, consider two random responses $r_1(t)$ and $r_2(t)$which were simultaneously recorded. To compute two auto RDD functions triggering conditions must be simultaneously applied to both measurements; thus:

$$x_1(\tau) = \Sigma\, r_1(t_i+\tau) \tag{4-a}$$
$$x_2(\tau) = \Sigma\, r_2(T_i+\tau) \tag{4-b}$$

Considering only the deterministic part of equation (1) and expressing that homogeneous solution in terms of modal properties then:

$$x_1(\tau) = \sum_{i=1}^{N}\sum_{j=1}^{m} a_{ij}\alpha_j e^{\lambda_j(\tau)} \tag{5-a}$$
$$x_2(\tau) = \sum_{i=1}^{N}\sum_{j=1}^{m} a_{ij}\beta_j e^{\lambda_j(\Delta t_i+\tau)} \tag{5-b}$$

where α_j and β_j are the normalized modal displacements of mode j in x_1 and x_2, a_{ij} is a constant for mode number j in ensemble number i and $\Delta t_i = T_i - t_i$

Rewriting equations (5),

$$x_1(\tau) = \sum_{j=1}^{m}\left(\sum_{i=1}^{N} a_{ij}\right)\alpha_j e^{\lambda_j\tau} \tag{6-a}$$
$$x_2(\tau) = \sum_{j=1}^{m}\left(\sum_{i=1}^{N} a_{ij}\, e^{\lambda_j\Delta t_i}\right)\beta_j\, e^{\lambda_j\tau} \tag{6-b}$$

As it can be seen from equation (6), the relative mode shape of measurement 2 relative to measurement 1 is:

$$\left(\phi_{2/1}\right)_j = \left(\sum_{i=1}^{N} a_{ij}e^{\lambda_j\Delta t_i} / \sum_{i=1}^{N} a_{ij}\right)(\beta_j/\alpha_j) \tag{7}$$

Needless to say, this is an erroneous mode shape and need to be corrected. Correction becomes possible in the following situations:

2.1 Zero Time Shift Consistent Sign Vector Triggering

This is the case where at triggering times t_i the sign of response vector remain unchanged (e.g. in the case of two responses the sign of x_1 and x_2 remains ++, +–, –+ or – –). In this case Δt_i is zero and the two RDD functions are auto functions.

2.2 Constant Time Shift Vector Triggering

In this case one is to seek a constant time shift, $(\Delta t_i = \Delta\tau)$, at which both measurement satisfy their individually imposed triggering conditions. This constant time shift is applied to all ensembles.

2.2.1 Choice of Time Shifts

For n measurements, the time shifts Δt_1, Δt_2, $\ldots\Delta t_n$ should be selected in a way to obtain the maximum number of trig points. The obvious possibility is to estimate a column in the covariance functions at both positive and negative time points using the normal RDD technique with one triggering measurement j. To obtain the maximum number of triggering points for measurement i the time shifts Δt_i can be chosen from

$$\max\left(x_{i,j}(\tau)\right) \Rightarrow \Delta t_i = \tau \tag{8}$$

The time shift corresponding to $i = j$ is $\Delta t_i = 0$ which is the time lag with maximum value for the auto covariance of a stationary time series.

2.2.2 Correction for Time Shifts

In order to take the time shift into account the procedure is that every VRDD function is shifted minus the corresponding time delay used in the triggering condition on the actual measurement. Then a number of points equal to the largest magnitude of the time shifts used in the measurement set is deleted.

3. SIMULATED EXPERIMENTS

3.1 Experiment 1:

This experiment is based on a 2–DOF system with the following modal parameters.

f [Hz]	ζ [%]	$\lvert\Phi\rvert^1$	$\lvert\Phi\rvert^2$	$\angle\Phi^1$	$\angle\Phi^2$
2.0984	0.0109	1.0000	1.4757	0.0000	178.88
1.2945	0.0339	1.0000	0.6775	0.0000	0.68

The results are based on 200 independent simulations. The system is loaded with uncorrelated white noise at each mass, and simulated with an ARMAV model which assures covariance equivalence between the continuous and discrete response. Furthermore, because the system is so simple 10% independent Gaussian white noise are added to both responses (10% is standard deviation of the noise divided by the standard deviation of the noise free response). Five hundred points are generated in each time series. The sampling frequency is 10 Hz.

The random responses were processed using different types of RDD triggering. The resulting RDD functions are then used in the ITD modal identication algorithm, [8].

Three different quality measures are used: Bias, equation (9), Variance, equation (10) and Relative Error, equation (11). In all three equations x could be the theoretical frequency, damping ratio or magnitude of the non-unit normalized mode shape component. $\hat{x}$ is the corresponding estimate from RDD-ITD

$$Bias = \sum_{i=1}^{2} \frac{|x_i - \hat{x}_i|}{\sigma_{x_i}} \qquad (9)$$

$$Variance = \sum_{i=1}^{2} \frac{\sigma_{x_i}}{x_i} \qquad (10)$$

$$Relative\ Error = \sum_{i=1}^{2} \frac{|x_i - \hat{x}_i|}{x_i} \qquad (11)$$

Figure 1 shows the RDD functions using triggering at a single measurement. The time lags is positive and negative. The (*) and the (o) marked on the cross RDD functions give optimum time delays for vector triggering as discussed in Section 2.2.1

Figure 2 shows the typical VRDD functions with both positive and negative time. Notice that the functions are not symmetric around $\tau = 0$, which indicates that the VRDD functions are not pure auto correlation functions.

For the 200 simulations the average number of trig points were:

RDD1	RDD2	VRDD(*)	VRDD(o)
200	200	145	55

The above table illustrates the influence of the triggering time shifts on the expected number of triggering points. The quality measures are calculated from the 200 simulations. The results are shown in Figure 3. Five different bars are shown in each figure. Bar 1 corresponds to results from using RDD functions using triggering at response of the first mass only. Bar 2 corresponds to results from using RDD functions estimated from triggering at the response of the second mass only. Bar 3 corresponds to results from all 4 RDD functions. Bar 4 corresponds to results from VRDD functions estimated using a time shift corresponding to (*) at the response of the second mass. Bar 5 corresponds to results from VRDD functions estimated using a time shift corresponding to (o) at the response of the second mass.

The results indicate the importance of choosing time shifts for triggering with high correlation. The figures show that the VRDD technique, Bar 4, can be more accurate than the traditional RDD technique. The results in VRDD Bar 5 show the effects of using improper time delay and low number of averages.

3.2 Experiment 2

This experiment is based on a 2–DOF system with the following modal parameters.

f [Hz]	ζ [%]	$\lvert\Phi\rvert^1$	$\lvert\Phi\rvert^2$	$\angle\Phi^1$	$\angle\Phi^2$
1.5583	0.0219	1.0000	0.0829	0.0000	2.2114
4.2231	0.0183	1.0000	12.0246	0.0000	174.0182

The results are based on 200 independent simulations. The system are loaded by uncorrelated white noise at each mass, and simulated with an ARMAV model which assures covariance equivalence between the continuous and discrete response. 2000 points are generated in each time series. The sampling frequency is 20 Hz. Figure 4 shows the RDD functions using triggering at a single measurement

Figure 5 shows the typical VRDD functions with both positive and negative time. the time shifts is chosen to $1 \cdot \Delta t$, which corresponds to maximum correlation between the two measurements as seen from Figure 4. The average number of triggering points were:

RDD 1	RDD 2	VRDD
960	960	500

The RMS of the RDD functions in figure 4 and figure 5 are:

RDD 11	RDD 21	RDD 12	RDD 22	VRDD1	VRDD2
0.5767	0.0678	0.0620	0.4606	0.5871	0.4365

Figure 6 shows the results for the quality measures as defined in equations (9) to (11). Bar 1 is the results from triggering at the response of the 1st mass, Bar 2 is the results from triggering at the response of the 2nd mass, Bar 3 is the results from all 4 RDD functions (see figure 4) and Bar 4 is the results from the VRDD technique.

Figure 6 shows that the VRDD approach is superior compared to the RDD technique where only triggering at a single measurement is used.

4. ACKNOWLEDGEMENTS

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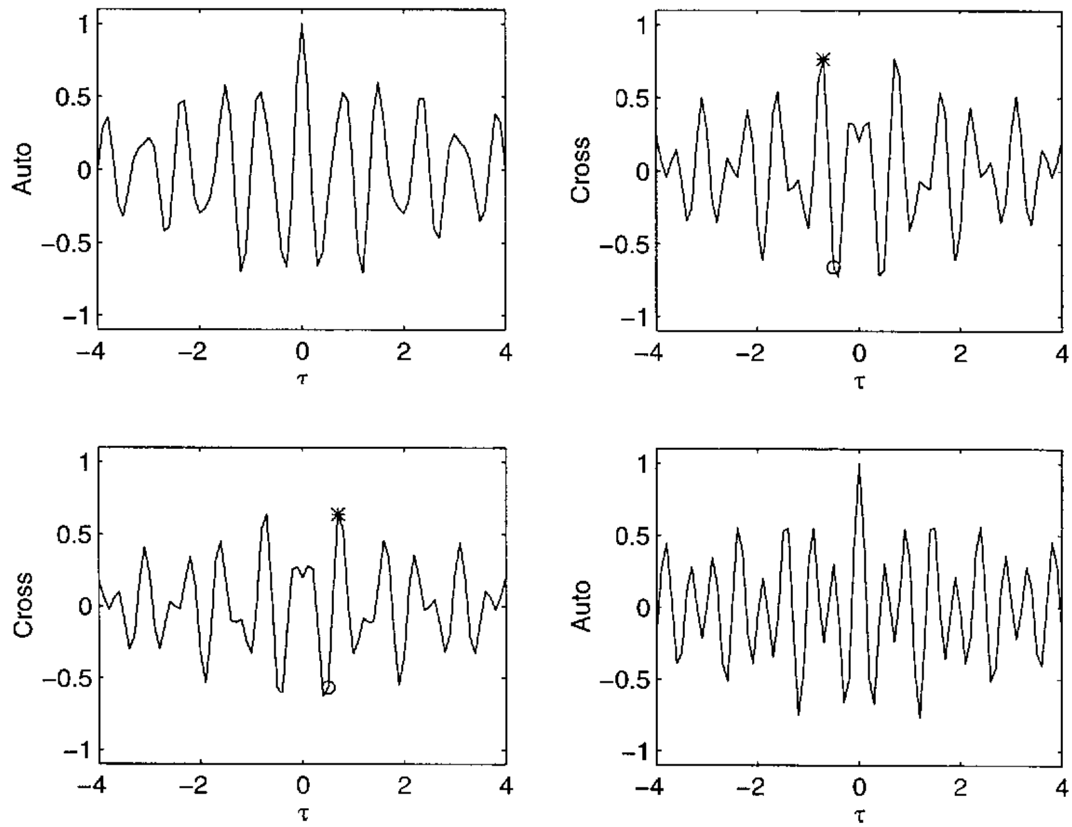

Figure 1. RDD functions using measurement 1 (left) and measurement 2 (right) for triggering. The (*) and (o) on the cross functions designate optimum time delays for VRDD calculations

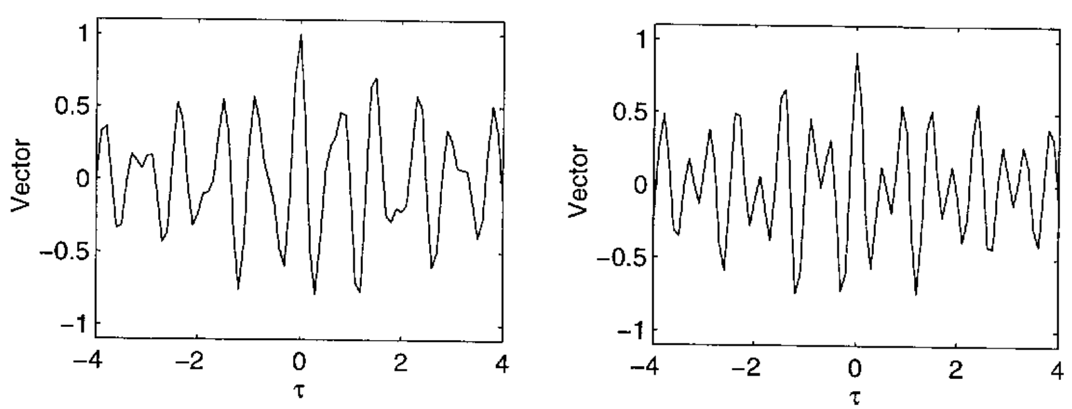

Figure 2. VRDD functions using the (*) time delay from Figure 1.

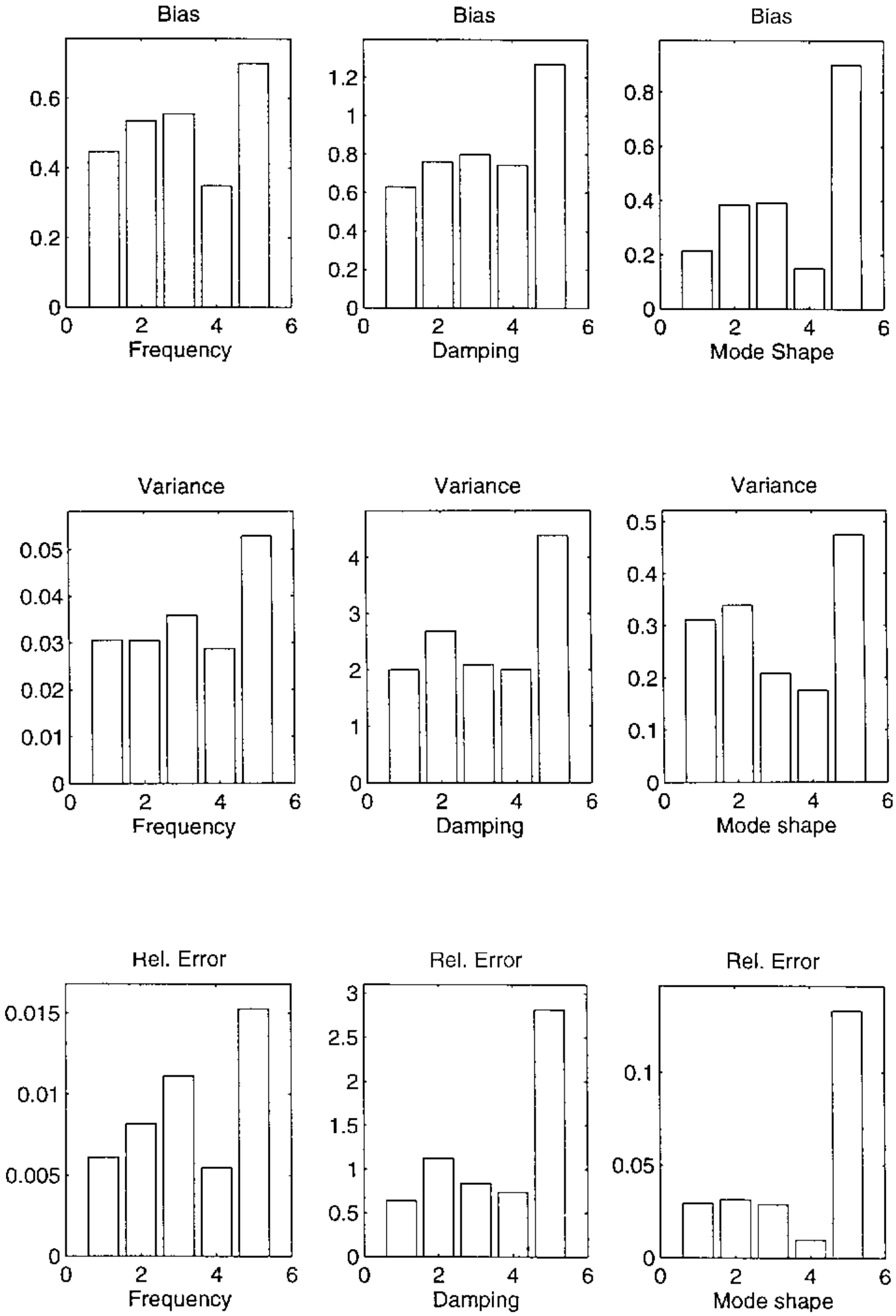

Figure 3. Bias, Variance and Relative Error for different triggering conditions for experiment 1.

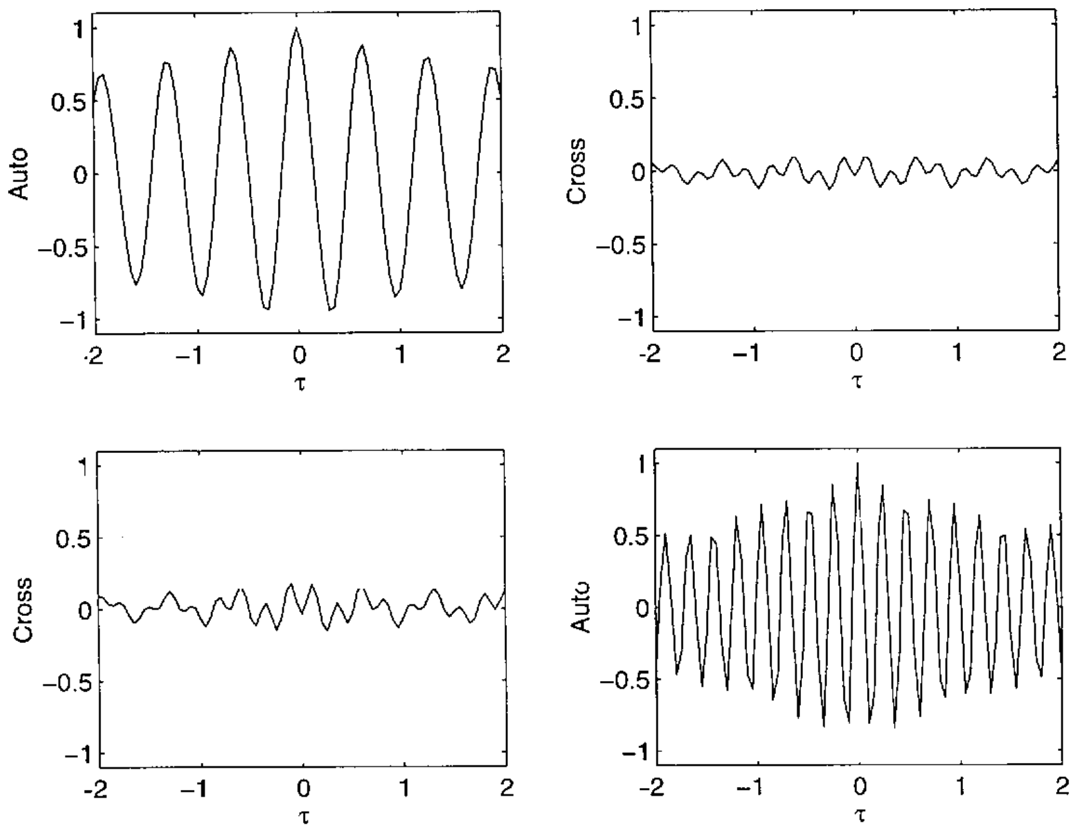

Figure 4. RDD functions using measurement 1 (left) and measurement 2 (right) for triggering.

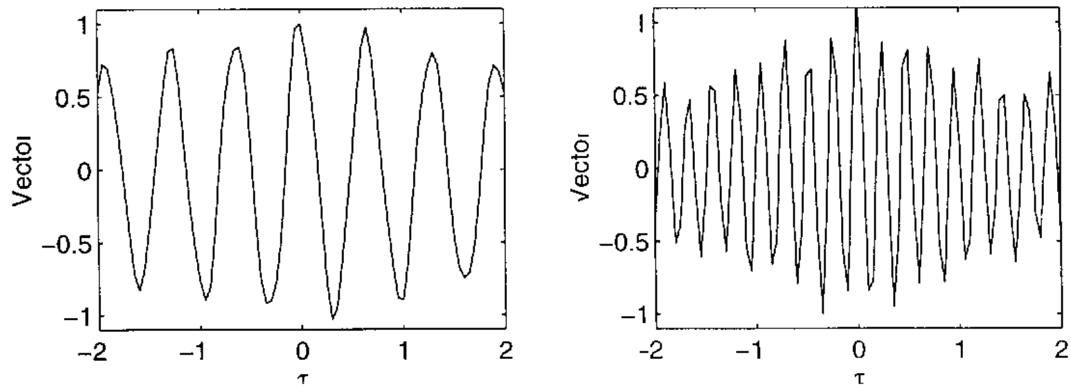

Figure 5. VRDD functions for experiment 2

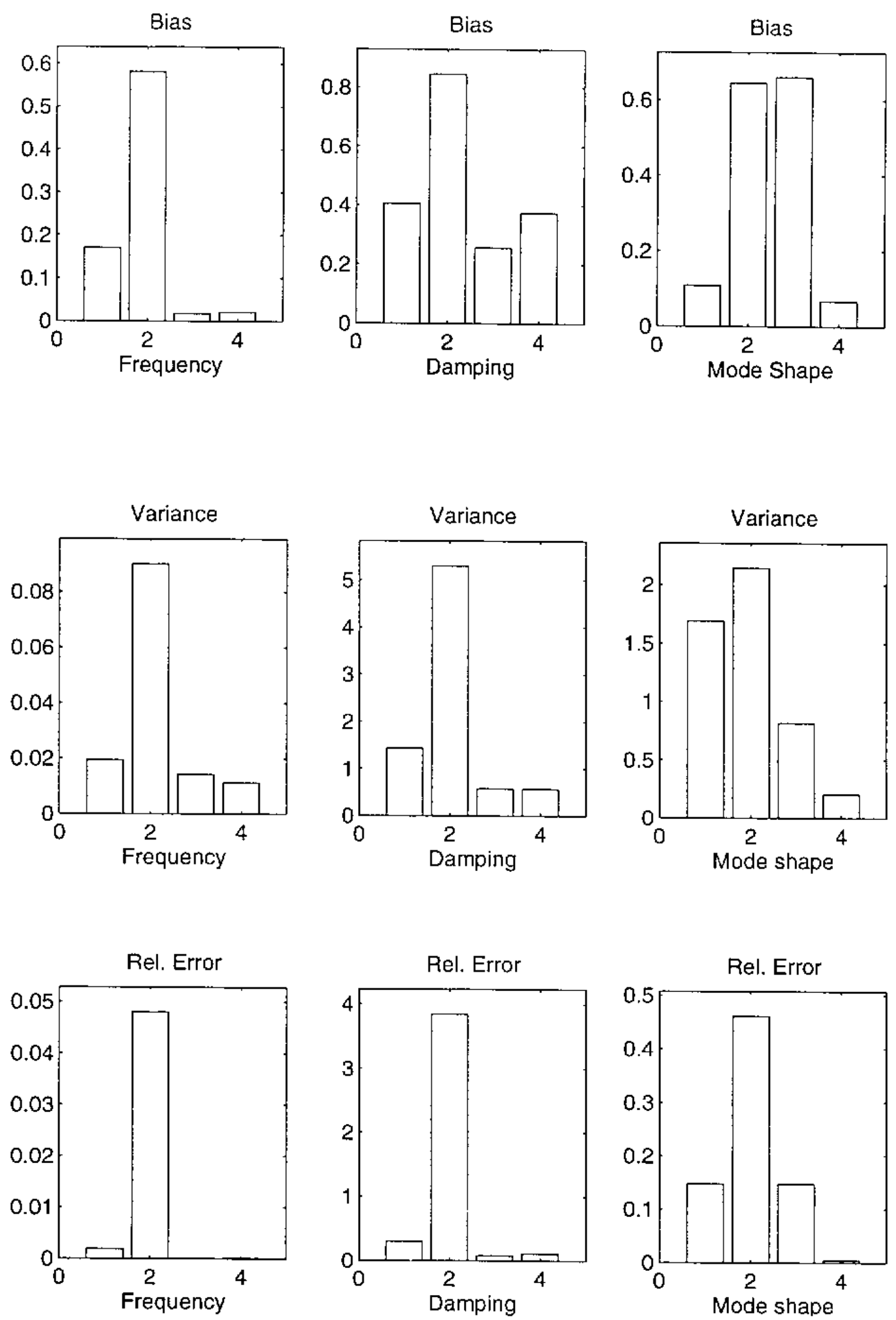

Figure 6. Bias, Variance and Relative Error for different triggering conditions for experiment 2.