

FILTERING OUT ENVIRONMENTAL EFFECTS IN DAMAGE DETECTION OF CIVIL ENGINEERING STRUCTURES

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ABSTRACT

This paper concerns the problems of using eigen-frequencies, estimated from sampled data, of a structural system exposed to fluctuating ambient conditions. In this paper it is the effects of a fluctuating ambient temperature that is of primary concern. A regression model, for elimination of the influence of the ambient temperature on the estimated natural eigen-frequencies, is formulated. This regression model is tested using simulations of a steel beam, having a growing crack and being exposed to a fluctuating ambient temperature. It is found that even with a simple parametrization, it is possible to remove the influence of the fluctuating ambient temperature from the estimated eigen-frequencies.

NOMENCLATURE

$\alpha(t)$	Time-varying environmental dependent parameter
β	Temperature gradient
$\beta(t)$	Time-varying gain parameter
Φ	Damage parameter
$T(t)$	Time-dependent ambient temperature
T^0	Reference ambient temperature
$U(t)$	Row vector of time-dependent ambient conditions
U^0	Row vector of reference ambient conditions
$E(T)$	Temperature-dependent Young's module
$f(t, T(t))$	Time- and temperature-dependent eigen-frequency
$f(t, T^0)$	Time-dependent reference eigen-frequency
$v(t), w(t)$	White noise
$e(t)$	Innovations

1. INTRODUCTION

Damage assessment in civil engineering structures using vibration measurements is a problem which has received much attention recently. Such vibration based inspection techniques are particularly needed for dealing with large structures, such as civil engineering structures. The reason is that they do not need access to the structures for the investigator, such as those techniques based on e.g. visual inspec-

tion, ultrasonic testing, eddy currents and acoustic emission. Further, research in vibration based damage assessment techniques has been initiated by a considerable demand for a more accurate non-destructive damage assessment technique.

The most commonly applied vibration based damage assessment techniques are only based on changes of natural eigen-frequencies. By comparing changes of experimentally measured natural eigen-frequencies in structures with patterns of changes predicted theoretically, the location and/or the size of the damage can be obtained.

However, real damage assessment tests imply that the investigator has to distinguish between changes in natural eigen-frequencies, due to effects produced by damages, and those brought about as a result of changes in ambient conditions. The ambient conditions are for example; temperature, wind speed and direction, relative humidity, and pore pressure in the underlying soil.

Especially the changes in the natural eigen-frequencies, due to changes in the ambient temperature, have been reported by several researchers, such as Woon et al. [1], Roberts et al. [2], Askegaard et al. [3], Bencat [4], and Kirkegaard [5]. Woon et al. [1] has also studied the influence of the relative humidity on the change of the natural eigen-frequencies of a steel plate. It is concluded that the influence of changes in the relative humidity is insignificant compared to the influence of the changes of the temperature.

This paper presents a technique for filtering out the effect on the natural eigen-frequencies of a system, caused by changes in the ambient temperature. By filtering out the effects means bringing the eigen-frequencies to a state, described by a reference ambient temperature. To illustrate the ideas behind the approach, section two shows how an analytical regression model for the temperature dependency of a steel beam can be obtained. This approach is then generalized in sections three, four and five, by including a noise description. In section six the approach is tested using simulations of a steel beam with a growing crack, exposed to a fluctuating ambient temperature.

2. A CRACKED BEAM INFLUENCED BY A FLUCTUATING AMBIENT TEMPERATURE

Let t denote time and let $T(t)$ be the corresponding ambient temperature. The reference temperature that corresponds to the state, in which a description of the system is desired, will be denoted T^0 . This superscript will in what follows be used on every parameter that describes this basic state. Figure 1.A shows the natural eigen-frequency $f(t, T^0)$ of a bending mode of a simple supported beam. This eigen-frequency has, at the reference ambient temperature, a smooth decreasing trend due to the growing of a crack. However, it is not this behaviour that will be observed when the beam is influenced by a fluctuating ambient temperature $T(t)$ of the kind shown in figure 1.B. In this case the decrease of the eigen-frequency $f(t, T(t))$ is overlaid by fluctuations of the same kind as the temperature shown in figure 1.C. In this figure the eigen-frequency $f(t, T^0)$ is plotted as well. It should also be observed that, if the reference temperature is different from the expected value of the ambient temperature, then there will also be a constant difference between the eigen-frequency at the reference temperature and the eigen-frequency at the ambient temperature.

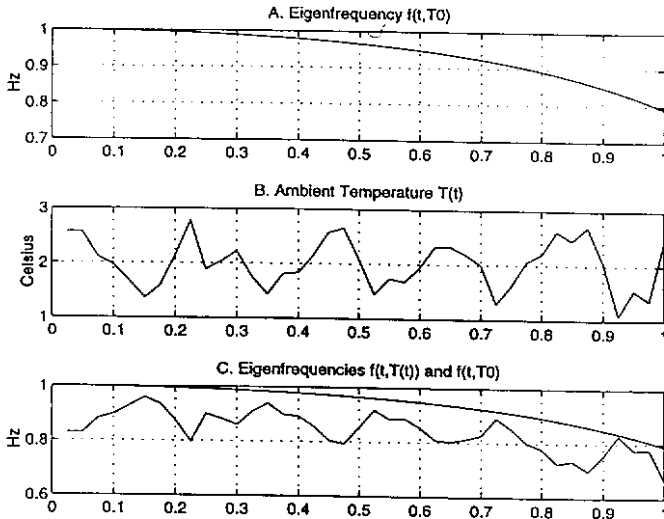


Figure 1: Influence of ambient temperature on an eigen-frequency of a beam with growing cracks.

The change in the ambient temperature will typically affect the geometry and the stiffness of the beam. If a crack at the same time is introduced, it will also affect the stiffness and the geometry of the component. The problem is how to separate the changes, originating from the damage, from the changes caused by the fluctuating ambient temperature.

Each of the bending modes of a beam has an eigen-frequency f_i that is described by the following expression

$$f_i = \frac{K_i}{l^2} \sqrt{\frac{EI}{\mu}} \quad (1)$$

where K_i is a constant, that describes the behaviour of the

mode, due to how the boundary conditions of the beam are. l is the length and μ is the mass per unit length of the beam. EI is the bending stiffness, with E being Young's modulus and I being the moment of inertia. In the following the subscript i will be dropped for notational reasons.

Assume that a crack has evolved for some time, causing changes in the moment of inertia. This change implies that the parameter is time-dependent. It will also be dependent on the temperature of the beam, due to thermal expansion effects. This temperature dependency also accounts for the length of the beam l .

It is fair to assume that Young's modulus is independent of the extent of the crack in the beam. On the other hand, observations show that it depends on the temperature. In Woon et al. [1] and Woon et al. [6] it is shown through experimental investigations that $E(T(t))$ depends linearly on the temperature. In other words

$$E(T(t)) = E(T^0) + \beta(T(t) - T^0) \quad (2)$$

where β is a temperature gradient of $E(T(t))$ that describes the linear change due to the temperature difference, and $E(T^0)$ is the Young's modulus at the reference ambient temperature.

These considerations imply that the eigen-frequency is described by the following relation

$$f(t, T(t)) = \frac{K}{l^2(T(t))} \sqrt{\frac{E(T(t))I(t, T(t))}{\mu}} \quad (3)$$

In Woon et al. [1] it is shown that the changes of both $l(T(t))$ and $I(t, T(t))$ are insignificant, compared to the change of $E(T(t))$. So, for simplicity, it is assumed that these parameters are insensitive to temperature changes. In other words, that $l(T(t)) = l$, and $I(t, T(t)) = I(t)$.

From (3) a relation between two different damage states, t_1 and t_2 , at the reference ambient temperature, can be obtained as

$$f(t_1, T^0)^2 = \frac{I(t_1)}{I(t_2)} f(t_2, T^0)^2 \quad (4)$$

$$= \Phi(t_1, t_2) f(t_2, T^0)^2$$

by insertion of t_1 and t_2 in (3), followed by a division. The two damage states are as such related through a time-varying parameter $\Phi(t_1, t_2)$

At time step t the following relation between $f(t, T(t))^2$ and $f(t, T^0)^2$ can be obtained from (2) and (3)

$$\begin{aligned}
f(t, T(t))^2 &= \frac{K^2}{I^4 \mu} E(T(t)) I(t) \\
&= \frac{K^2}{I^4 \mu} E(T^0) I(t) + \frac{K^2 I(t) \beta}{I^4 \mu} (T(t) - T^0) \quad (5) \\
&= f(t, T^0)^2 + \alpha(t) (T(t) - T^0)
\end{aligned}$$

with $\alpha(t)$ defined as

$$\alpha(t) = \frac{K_i^2 I(t) \beta}{I^4 \mu} \quad (6)$$

Therefore, knowing $f(t, T(t))$ and $\alpha(t)$, the eigen-frequency is at the reference ambient temperature given by

$$f(t, T^0)^2 = f(t, T(t))^2 - \alpha(t) (T(t) - T^0) \quad (7)$$

From (4) and (7) it is possible to derive a regression model for filtering out the effects of fluctuating ambient temperatures. This can be simplified if (6) is formulated in terms of a known sequence instead of $I(t)$. The expression for $\alpha(t)$ can be rewritten by inserting (3), at the reference ambient temperature, into (6) as

$$\alpha(t) = \frac{K^2 I(t) \beta}{I^4 \mu} = \frac{\beta}{E(T^0)} f(t, T^0)^2 \quad (8)$$

The unknown eigen-frequency $f(t, T^0)^2$ must be substituted by some known quantities. This can be accomplished by inserting (7) into (8) to yield

$$\begin{aligned}
\alpha(t) &= \frac{\beta}{E(T^0)} (f(t, T(t))^2 - \alpha(t) (T(t) - T^0)) \\
&= \frac{\frac{\beta}{E(T^0)} f(t, T(t))^2}{1 + \frac{\beta}{E(T^0)} (T(t) - T^0)} \quad (9)
\end{aligned}$$

$\alpha(t)$ is now expressed in terms of the two known sequences $f(t, T(t))$ and $T(t)$, and the regression model can now be formulated on the basis of (4), (7) and (9).

Define $t_1 = t$ and $t_2 = t-1$, equation (4) is then given by

$$f(t, T^0)^2 = \Phi(t, t-1) f(t-1, T^0)^2 \quad (10)$$

Inserting (7) into (10) yields

$$\begin{aligned}
f(t, T(t))^2 - \alpha(t) (T(t) - T^0) &= \\
\Phi(t, t-1) f(t-1, T(t-1))^2 - & \\
\Phi(t, t-1) \alpha(t-1) (T(t-1) - T^0) & \quad (11)
\end{aligned}$$

which is a regression model for $f(t, T(t))^2$, given temperatures $T(t)$, and given $f(t-n, T(t-n))^2$, for $n = t-1$ down to $n = t_0$. Since $f(t, T(t))$ and $T(t)$ are known in advance, it is in principle possible to estimate $\Phi(t, t-1)$ and $\alpha(t)$ from (10). Having estimated $\Phi(t, t-1)$ and $\alpha(t)$, an estimate of the eigen-frequencies at the reference conditions can be obtained from

$$\hat{f}(t, T^0) = \sqrt{f^2(t, T(t)) - \alpha(t) (T(t) - T^0)} \quad (12)$$

If the model is to be used for estimated eigen-frequencies, it has to be possible to take into account the presence of noise, originating from estimation inaccuracies. The regression model must also be more general, so it can be used for other structural components.

3. A GENERAL REGRESSION APPROACH

In the previous section the effects of fluctuating ambient temperatures were considered in a deterministic way for a steel beam. As stated at the end of that section, there are a number of problems that cannot be solved by the above regressions model. However, the derived regression model can be used as guideline for the development of a more general regression model. In most cases the evolution of the eigen-frequency of the structure, due to increasing damages in a structure, is not described by a smooth curve as in figure 1.A. There will most certainly be an unknown random influence that is independent of the fluctuations of the ambient temperature. This behaviour is illustrated in figure 2.A for an eigen-frequency, at a reference ambient temperature T^0 , of a system influenced by a fluctuating temperature $T(t)$. The observations of $T(t)$ are shown in figure 2.B. The influence of $T(t)$ and the unknown random component, on the eigen-frequency, are illustrated in figure 2.C, together with the eigenfrequency at the reference ambient temperature.

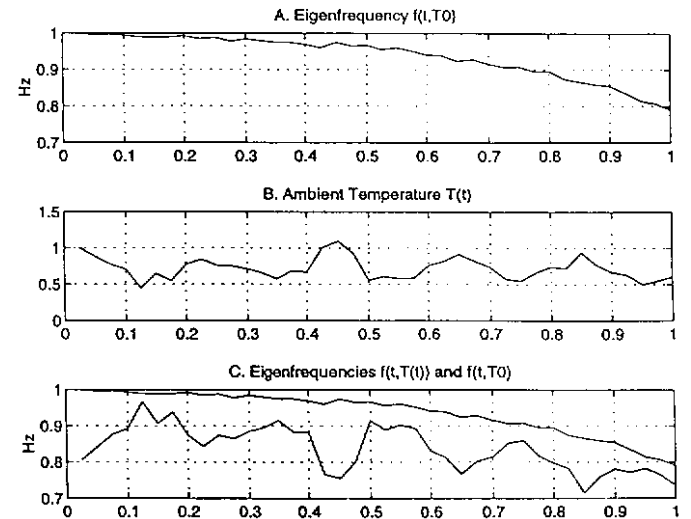


Figure 2: Influence of an observable ambient temperature variation on an eigen-frequency of an increasingly damaged structure.

In the general modelling approach, it is not easy to make use of physical knowledge as it was in the previous section. So, from an empirical point of view, the simplest way of describing the filtered eigen-frequency, at reference ambient temperature T^0 , is by the following model

$$f(t+1, T^0) = \Phi f(t, T^0) + w(t) \quad (13)$$

where Φ is a constant parameter, and $w(t)$ is a residual sequence that is assumed to be a white noise. This relation both models a possible change of the eigen-frequency and the unpredictable random component.

It is, however, the eigen-frequency $f(t, T(t))$ that corresponds to the fluctuating ambient temperature that is known at this state. It is assumed that $f(t, T(t))$ is related to $f(t, T^0)$ by the following linear relation

$$f(t, T(t)) = f(t, T^0) + \alpha(t)(T(t) - T^0) + v(t) \quad (14)$$

with $\alpha(t)$ being a time-varying parameter, and $v(t)$ is a white noise sequence. Equations (13) and (14) constitute a discrete-time linear system of the form

$$\begin{aligned} f(t+1, T^0) &= \Phi f(t, T^0) + w(t) \\ f(t, T(t)) &= f(t, T^0) + \alpha(t)(T(t) - T^0) + v(t) \end{aligned} \quad (15)$$

From theory of the Kalman filtering of linear systems, see Goodwin et al. [7], the predictor of $f(t, T(t))$ is obtained from (15) as

$$\begin{aligned} \hat{f}(t+1, T^0) &= \Phi \hat{f}(t, T^0) + \beta(t)e(t) \\ \hat{f}(t, T(t)) &= \hat{f}(t, T^0) + \alpha(t)(T(t) - T^0) \end{aligned} \quad (16)$$

where $e(t)$ is a residual sequence, defined as

$$e(t) = f(t, T(t)) - \hat{f}(t, T(t)) \quad (17)$$

and $\beta(t)$ is a time-varying gain. From (17), and the second equation in (16), the predictor of $f(t, T^0)$ can be expressed as

$$\hat{f}(t, T^0) = f(t, T(t)) - \alpha(t)(T(t) - T^0) - e(t) \quad (18)$$

By introducing the backward shift operator, defined by the relation $q^{-1}y(t) = y(t-1)$, and inserting (18) into the first equation in (16) yields the following model for $f(t, T(t))$

$$f(t, T(t)) = \alpha(t)(T(t) - T^0) + \frac{1 - (\Phi - \beta(t))q^{-1}}{1 - \Phi q^{-1}} e(t) \quad (19)$$

in which the unknown eigen-frequency $f(t, T^0)$ is eliminated. Following Ljung [8] the one-step ahead predictor of (19) is given by

$$\hat{f}(t, T(t)) = \frac{1 - \Phi q^{-1}}{1 - (\Phi - \beta(t-1))q^{-1}} \alpha(t)(T(t) - T^0) + \frac{\beta(t-1)q^{-1}}{1 - (\Phi - \beta(t-1))q^{-1}} f(t, T(t)) \quad (20)$$

The question is how to parametrize $\alpha(t)$ and $\beta(t)$. This question has to be answered before the predictor of the model can be used for estimation of $f(t, T^0)$. Having answered this question, a non-linear least-squares estimate of the parameters can then be obtained by minimizing a criterion function of the form

$$V(\Phi, \alpha(t), \beta(t)) = \sum_{t=1}^N (f(t, T(t)) - \hat{f}(t, T(t)))^2 \quad (21)$$

This minimization can be performed using e.g. a Gauss-Newton search scheme, see Ljung [8].

The procedure for removal of the influence of the ambient temperature on the eigen-frequency estimates can be summarized as follows.

- A. Obtain N estimates of the eigenfrequency $f(t, T(t))$, and record the corresponding ambient temperature $T(t)$, from long term observations of a structure.
- B. Define a reference ambient temperature T^0 , and minimize the criterion function (21), with a suitable parametrization of $\alpha(t)$ and $\beta(t)$.
- C. From the estimated parameters, an estimate of the eigen-frequencies $f(t, T^0)$, at the reference ambient temperature, can be obtained from (18).

4. PARAMETRIZATION OF $\alpha(t)$ AND $\beta(t)$

There are a number of ways to parametrize $\alpha(t)$ and $\beta(t)$. This section will describe two simple parametrizations. First of all, let $\alpha(t)$ depend linearly on the expected value of the eigen-frequency at the reference ambient temperature

$$\alpha(t) = \alpha_1 E[\hat{f}(t, T^0)] \quad (22)$$

This will be a good approximation if the disturbance of the eigen-frequency $f(t, T^0)$, due to the fluctuating ambient temperature, is small. However, since $f(t, T^0)$ is unknown, its expected value will be substituted using (14), implying

$$\begin{aligned}
\alpha(t) &= \alpha_1 E[f(t, T(t)) - \alpha(t)(T(t) - T^0) - v(t)] \\
&= \alpha_1 (f(t, T(t)) - \alpha(t)(T(t) - T^0)) \\
&= \frac{\alpha_1 f(t, T(t))}{1 + \alpha_1 (T(t) - T^0)}
\end{aligned} \tag{23}$$

which is a non-linear function of the parameter α_1 .

From theory of Kalman filtering, see e.g. Goodwin et al. [7], the gain $\beta(t)$ can be written as

$$\beta(t) = \frac{\Phi E[(f(t, T^0) - \hat{f}(t, T^0))^2]}{E[(f(t, T^0) - \hat{f}(t, T^0))^2] + E[v^2(t)]} \tag{24}$$

where $E[v^2(t)]$ is the variance of the white noise sequence $v(t)$. If the estimates of $f(t, T(t))$, used in the regression model, are very accurate, it is fair to assume that the noise term $v(t)$ will approach zero. Then from (24) it follows that $\beta(t) = \Phi$, and that (20) can be written as

$$\begin{aligned}
\hat{f}(t, T(t)) &= (1 - \Phi q^{-1})\alpha(t)(T(t) - T^0) + \\
&\quad \Phi f(t-1, T(t-1))
\end{aligned} \tag{25}$$

with $\alpha(t)$ defined in (23). However, this simple parametrization does not remove the influence of the noise $w(t)$. Removal of this noise from $f(t, T^0)$ can only be accomplished by a re-parametrization of $\beta(t)$.

5. INFLUENCE OF OTHER AMBIENT CONDITIONS

The general regression approach may of course also be used for filtering out effects of other ambient conditions influencing the eigen-frequencies. The only requirement is that (25) is fulfilled by a proper choice of $\alpha(t)$.

The eigenfrequency can also be influenced by several fluctuating ambient conditions. Denote these by $U(t)$, defined as $U(t) = \{u_1(t), u_2(t), \dots, u_n(t)\}$, and define the corresponding reference conditions as $U^0 = \{u_1^0, u_2^0, \dots, u_n^0\}$. If they are mutually independent then (25) can be extended as

$$\begin{aligned}
f(t+1, U^0) &= \Phi f(t, U^0) + w(t) \\
f(t, U(t)) &= f(t, U^0) + \sum_{i=1}^n \alpha_i(t) (u_i(t) - u_i^0) + v(t)
\end{aligned} \tag{26}$$

with a corresponding predictor of $f(t, U(t))$ looking like

$$\begin{aligned}
\hat{f}(t, U(t)) &= \frac{1 - \Phi q^{-1}}{1 - (\Phi - \beta(t-1))q^{-1}} \sum_{i=1}^n \alpha_i(t) (u_i(t) - u_i^0) + \\
&\quad \frac{\beta(t-1)q^{-1}}{1 - (\Phi - \beta(t-1))q^{-1}} f(t, T(t))
\end{aligned} \tag{27}$$

It then remains to parametrize the $\alpha_i(t)$'s and $\beta(t)$. In order to do this it is necessary to investigate how each of the fluctuating ambient conditions influence the eigen-frequency.

6. EXAMPLE

The applicability of the proposed filtering approach, with the simple parametrization of $\alpha(t)$ and $\beta(t)$, is applied to a simulated change of the bending natural eigen-frequency $f(t, T^0)$ of the lowest mode of a free-free steel beam. The change of this, at a reference temperature of $T^0 = 20^\circ\text{C}$, is shown in figure 3. During the simulations, the system has been exposed to the fluctuating ambient temperature shown in figure 4.

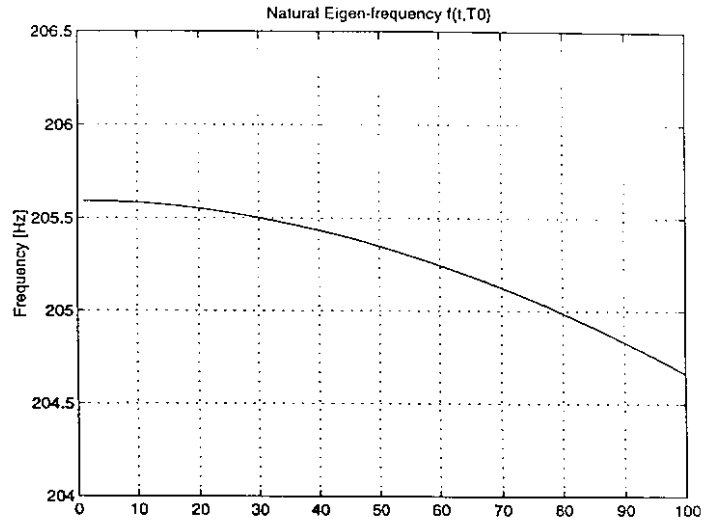


Figure 3: Simulated change of $f(t, T^0)$ of the lowest bending mode of a beam, at the reference ambient temperature.

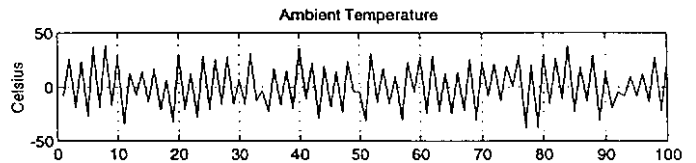


Figure 4: Ambient temperature variation used during the simulations.

The beam is 0.8 m long with a 0.025×0.025 m square cross section. The change of the bending eigen-frequency due to a damage has been estimated by an FEM of the beam. A damage in the beam is modelled by a fracture mechanical model. The development of a crack at a certain location of a beam corresponds to a sudden reduction of its bending stiffness at the same location. The crack divides the original

non-cracked beam into two shorter beams, connected, at the crack location, by a very infinitesimal portion of beam with different characteristics. The characteristic in the bending mode can be modelled by a torsion spring. Estimation of the spring stiffness to by fracture mechanics has been used by several authors, see e.g. Kirkegaard et al. [9] and Araújo Gomes et al. [10]. The fracture mechanical model, used in this paper, is reliable for maximal crack depth at 60% of the beam height. The FEM has been calibrated using experimental data from a non-cracked laboratory beam. This calibration has been performed to secure that the FEM describes the beam in the best possible way, see Kirkegaard et al. [8]

The temperature dependency of Young's modulus has been modelled according to (2), with $E(T^0) = 2.058 \times 10^{11} \text{ N/m}^2$, and a temperature gradient $\beta = 0.001 \text{ }^\circ\text{C}^{-1} \cdot E(T^0)$. The changes of the dimensions of the beam, due to thermal expansion effects, have been disregarded.

Two cases will be considered in the following. In both cases 100 simulations of the eigenfrequency has been used. The first case (case 1) will be noise-free. In other words, use the above simulations directly. If the general regression approach is consistent, it should be possible to estimate the eigenfrequency, at the reference ambient temperature, very accurately. In the second case (case 2) there has been added zero-mean white noise, with a standard deviation of $0.005 \cdot f(t, T(t))$, to each simulation $f(t, T(t))$. This violates the assumption that $v(t)$ is zero. It is therefore interesting to see the applicability of the simple parametrization in this case.

The temperature dependent eigen-frequency of case 1 is shown in figure 5, and for case 2 in figure 6.

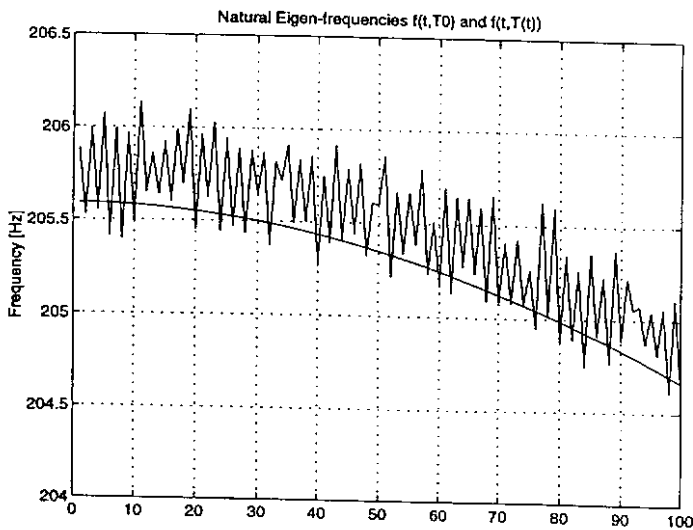


Figure 5: Case 1 - Simulated change of $f(t, T(t))$ of the lowest bending mode of a beam, influenced by a fluctuating ambient temperature. $f(t, T(t))$ is plotted together with $f(t, T^0)$.

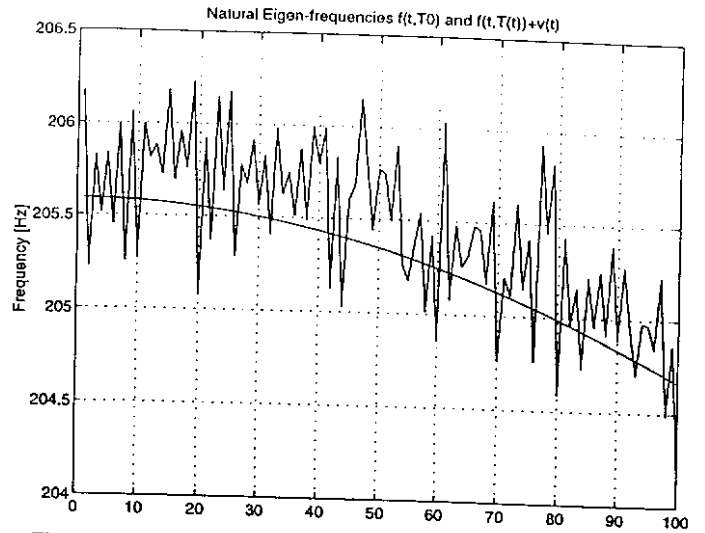


Figure 6: Case 2 - Simulated change of $f(t, T(t))$ of the lowest bending mode of a beam, influenced by a fluctuating ambient temperature. $f(t, T(t))$ is plotted together with $f(t, T^0)$.

Estimates of Φ and α_1 are obtained using the Gauss-Newton search scheme. These parameter estimates are, for both cases, together with their estimated standard deviations, listed in table 1.

Case	Φ	σ_Φ	α_1	σ_{α_1}
1	0.999	2.45×10^{-4}	-4.99×10^{-5}	5.79×10^{-8}
2	0.999	1.30×10^{-4}	-5.12×10^{-5}	3.08×10^{-6}

Table 1: Estimated parameters Φ and α_1 , and estimated standard deviations, for the eigen-frequency.

Finally, the eigen-frequency $f(t, T^0)$, is in both cases compared with the simulated one of figure 3.

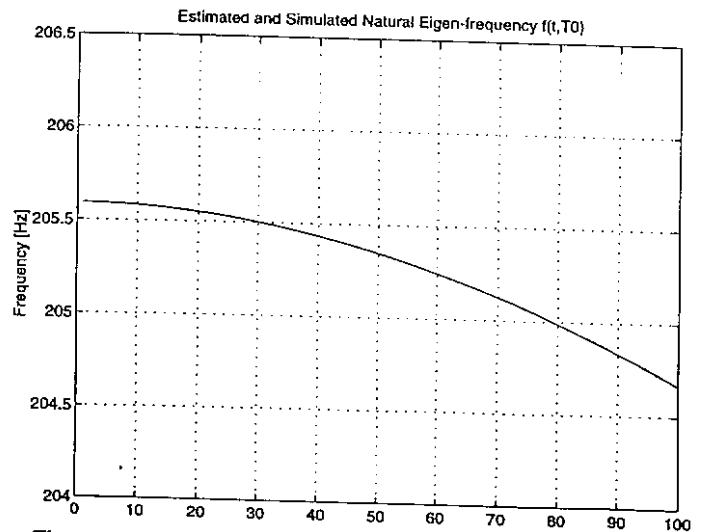


Figure 7: Case 1 - Comparison of estimated and simulated eigen-frequency $f(t, T^0)$, at the reference ambient temperature.

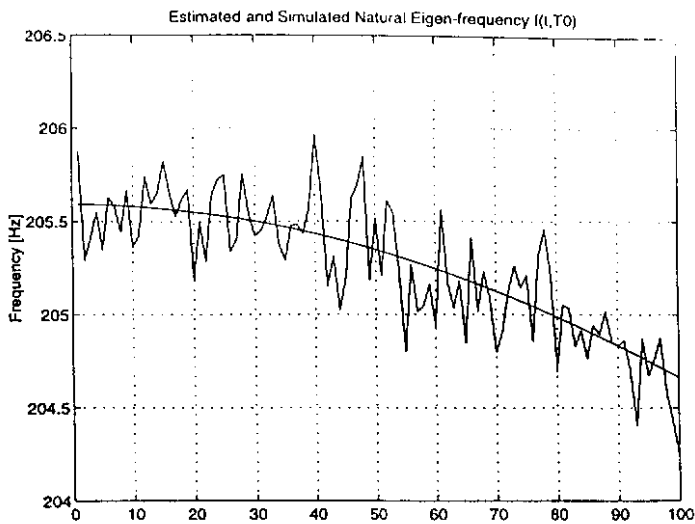


Figure 8: Case 2 - Comparison of estimated and simulated eigen-frequency $f(t, T^0)$, at the reference ambient temperature.

It is seen from figure 7 that in the noise-free case there is a complete match between the simulated and estimated eigen-frequency, at the reference ambient temperature. In the case where noise is present, it is not possible to remove the influence of this noise, and the fluctuating ambient temperature, completely. As stated before, this is of no surprise, since adding of noise is a clear violation of the assumption that the noise $v(t)$ is zero. However, the results indicates that with a proper parametrization of $\alpha(t)$ and $\beta(t)$, it is possible to remove the influence of the ambient temperature.

7. CONCLUSIONS

In this paper, it has been illustrated, how to remove the influence of fluctuating ambient temperature from estimates of natural eigen-frequencies of a damaged structure. In other words, how to put the estimates of the natural eigen-frequencies to a state, described by a reference ambient temperature.

As guideline for the development of a general regression model, an analytical model has been established for a steel beam having a temperature dependent Young's module. Based on this model, a general regression approach for filtering out the influence of a fluctuating ambient temperature, has been derived. The efficiency of this approach is highly dependent upon the parametrization of two time-varying parameters $\alpha(t)$ and $\beta(t)$.

The approach is tested on simulated change of the eigen-frequency of the lowest bending mode of a steel beam. If significant estimation errors are present in the eigen-frequency estimates used, the model cannot, with a simple parametrization of $\alpha(t)$ and $\beta(t)$, remove the influence of the fluctuating ambient temperature. In this case another parametrization of $\alpha(t)$ and $\beta(t)$ is necessary.

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