A NEW APPROACH FOR PREDICTING THE VARIANCE OF RANDOM DECREMENT FUNCTIONS

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Abstract *The Random Decrement (RD) technique is a simple and fast method for estimating the correlation functions of Gaussian processes. The RD functions are estimated as an averaging process in the time domain, which makes the technique simple and fast. If the RD technique is applied to stationary zero mean Gaussian distributed processes the RD functions are proportional to the correlation functions of the processes. If a linear structure is loaded by Gaussian white noise the modal parameters can be extracted from the correlation functions of the response, only. One of the weaknesses of the RD technique is that no consistent approach to estimate the variance of the RD functions is known. Only approximate relations are available, which can only be used under special conditions. The variance of the RD functions contains valuable information about the accuracy of the estimates. Furthermore, the variance can be used as basis for a decision about how many time lags from the RD functions should be used in the modal parameter extraction procedure. This paper suggests a new method for estimating the variance of the RD functions. The method is consistent in the sense that the accuracy of the approach is not dependent on neither the physical system nor the actual formulation of the RD technique.*

Nomenclature

a_1, a_2, b_1, b_2	Triggering levels.
$\mathrm{Cov}[\cdot\|\cdot]$	Covariance of conditional variables.
$D_{XX}, \dot{D}_{XX}$	RD function.
$\mathrm{E}[\cdot\|\cdot]$	Conditional mean value.
i, j	Integers.
N	Number of triggering points.
$p_X(x)$	Density function of Y.
R_{XX}, R_{YX}	Correlation functions.
R'_{XX}, R'_{YX}	Time derivative of R_{XX}, R_{YX}.
t, τ	Time variables.
$T^{G_A}_{X(t)}$	Applied general triggering condition.
$T^L_{X(t)}$	Level crossing triggering condition.
$T^P_{X(t)}$	Positive point triggering condition.
$\mathrm{Var}[\cdot\|\cdot]$	Variance of conditional variable.
X, Y	Ergodic stochastic processes.
$\dot{X}, \dot{Y}$	Time derivative of X and Y.
x, y	Realizations/observations of X, Y
σ_X	Standard deviation of X.

1 Introduction

The RD technique is a method to transform realizations of stochastic processes into so-called RD functions. The technique is usually applied to response measurements of structures, where the forces are unmeasurable, e.g. ambient. The basic idea behind the technique is to pick out time segments and average them each time the realizations fulfil a given initial condition, denoted a triggering condition. The estimation process of RD functions only involves detection of the triggering points and the averaging of the time segments, which makes the technique very fast. The technique was introduced by Cole, see Cole [1], [2], [3] and [4]. He used the technique for estimation of damping ratios and eigenfrequencies from single measurements and damage detection based on the RD functions. The fundamental idea behind extracting damping ratios and eigenfrequencies is that the RD functions are interpreted as free decays. Ibrahim, see Ibrahim [5] and [6], extended the theory and the application of the RD technique by introducing the concept of cross and auto RD functions in combination with the ITD algorithm. This extension made it possible to estimate mode shapes from the RD functions from multiple measurements.

In 1982 the theoretical background for the RD technique was extended by Vandiver et al. [7]. They proved that the RD function of a zero mean stationary Gaussian distributed stochastic process is proportional to the auto correlation function. Vandiver et al. also suggested an approximate method to estimate the variance of RD functions by assuming that the time segments in the estimation process are uncorrelated. The link between the RD functions and the correlation functions and the approximate method to predict the variance of RD func-

tions was extended by Brincker et al. [8], [9] and [10] to include the cross RD functions defined from the theoretical general triggering condition. An overview of these developments and further extension of these results to an applied general triggering condition can be seen in Asmussen [11]

The purpose of this paper is to investigate the variance of RD functions and to evaluate the existing method for estimation of the variance of RD functions. The variance of RD functions contains important information, since it can be used in the modal parameter extraction procedure. The main result is the introduction of a new approach to predict the variance of the RD functions, which takes the correlation between the time segments into account. The new approach is tested by a simulation study of different SDOF and 2DOF systems loaded by white noise. The variance of the RD functions is simulated and compared with the prediction of the two approaches

2 The Random Decrement Technique

Consider two stationary stochastic processes $X(t)$ and $Y(t)$. The auto, $D_{XX}(\tau)$, and cross, $D_{YX}(\tau)$, RD functions are defined as the mean value of $X(t)$ and $Y(t)$ on condition of $X(t)$

$$D_{XX}(\tau) = E[X(t+\tau)|T_{X(t)}] \tag{1}$$

$$D_{YX}(\tau) = E[X(t+\tau)|T_{X(t)}] \tag{2}$$

where $T_{X(t)}$ is denoted the triggering condition. The time variable τ can be both positive and negative and the triggering condition defines a set of initial conditions at time lag zero. For a multiple number of measurements, say n, n different sets of RD functions can be defined. A set of RD functions is the RD functions estimated on the basis of the triggering condition fulfilled at the same measurement. For e.g. $X(t)$ and $Y(t)$ D_{XX}, D_{YX}, and D_{XY}, D_{YY} constitutes two different sets of RD functions. The total number of RD functions is exactly equal to the total number of correlation functions, n^2.

By assuming that the stochastic processes are ergodic, the RD functions can be estimated as

$$\hat{D}_{XX}(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(t_i + \tau)|T_{x(t_i)} \tag{3}$$

$$\hat{D}_{YX}(\tau) = \frac{1}{N}\sum_{i=1}^{N} y(t_i + \tau)|T_{x(t_i)} \tag{4}$$

where $x(t)$ and $y(t)$ are realizations of $X(t)$ and $Y(t)$, which are assumed to be ergodic. The estimates are unbiased.

Several different triggering conditions exist. They can all be described from the applied general triggering condition, $T_{X(t)}^{GA}$, see Asmussen [11]

$$T_{X(t)}^{GA} = \{a_1 \le X(t) < a_2, \ b_1 \le \dot{X}(t) < b_2\} \tag{5}$$

where a_1, a_2, b_1 and b_2 are the triggering levels. If $X(t)$ and $Y(t)$ are stationary zero mean Gaussian distributed processes the RD functions of the applied general triggering condition are a weighted sum of the correlation functions and the time derivative of the correlation functions

$$D_{XX}(\tau) = \frac{R_{XX}(\tau)}{\sigma_X^2}\cdot\tilde{a} - \frac{R'_{XX}(\tau)}{\sigma_{\dot{X}}^2}\cdot\tilde{b} \tag{6}$$

$$D_{YX}(\tau) = \frac{R_{YX}(\tau)}{\sigma_X^2}\cdot\tilde{a} - \frac{R'_{YX}(\tau)}{\sigma_{\dot{X}}^2}\cdot\tilde{b} \tag{7}$$

where the triggering levels $\tilde{a}$ and $\tilde{b}$ are given by the Gaussian density function and the triggering levels a_1, a_2, b_1 and b_2, see eq. (5)

$$\tilde{a} = \frac{\int_{a_1}^{a_2} x p_X(x)dx}{\int_{a_1}^{a_2} p_X(x)dx} \quad \tilde{b} = \frac{\int_{b_1}^{b_2} \dot{x} p_{\dot{X}}(\dot{x})d\dot{x}}{\int_{b_1}^{b_2} p_{\dot{X}}(\dot{x})d\dot{x}} \tag{8}$$

The variance of the estimate of the RD functions can be predicted by assuming that the time segments used in the averaging process are uncorrelated. For simplicity only the variance of the estimated cross RD functions are given, but the results are easily extended to auto RD functions by substituting $X(t)$ with $Y(t)$ or opposite

$$\mathrm{Var}(\hat{D}_{YX}(\tau)) \approx \frac{\sigma_Y^2}{N}\left(1 - \left(\frac{R_{YX}(\tau)}{\sigma_Y\sigma_X}\right)^2 - \left(\frac{R'_{YX}}{\sigma_Y\sigma_{\dot{X}}}\right)^2\right) +$$

$$\left(\frac{R_{YX}(\tau)}{\sigma_X^2}\right)^2\left(\frac{k_4}{k_1} - \left(\frac{k_2}{k_1}\right)^2\right) + \tag{9}$$

$$\left(\frac{R'_{YX}(\tau)}{\sigma_{\dot{X}}^2}\right)^2\left(\frac{k_5}{k_1} - \left(\frac{k_3}{k_1}\right)^2\right)$$

where the constants k_1, k_2, k_3, k_4 and k_5 are given by

$$k_1 = \int_{a_1}^{a_2}\int_{b_1}^{b_2} p_{X\dot{X}}(x,\dot{x})d\dot{x}dx \tag{10}$$

$$k_2 = \int_{a_1}^{a_2}\int_{b_1}^{b_2} x p_{X\dot{X}}(x,\dot{x})d\dot{x}dx \tag{11}$$

$$k_3 = \int_{a_1}^{a_2}\int_{b_1}^{b_2} \dot{x} p_{X\dot{X}}(x,\dot{x})d\dot{x}dx \tag{12}$$

$$k_4 = \int_{a_1}^{a_2} \int_{b_1}^{b_2} x^2 p_{X\dot{X}}(x, \dot{x}) d\dot{x} dx \tag{13}$$

$$k_5 = \int_{a_1}^{a_2} \int_{b_1}^{b_2} \dot{x}^2 p_{X\dot{X}}(x, \dot{x}) d\dot{x} dx \tag{14}$$

Usually in application of the RD technique special triggering conditions are used. The relation between the RD functions of these conditions and the correlation functions and the approximate solution for the variance of the estimate of the RD functions can be extracted from the results for the applied general triggering condition

In this paper the level crossing triggering condition, $T_{X(t)}^L$, and the positive point triggering condition, $T_{X(t)}^P$ are considered in the examples, but the theory is valid from all possible reformulations of the applied general triggering condition.

$$T_{X(t)}^L = \{X(t) = a\} \tag{15}$$

$$T_{X(t)}^P = \{a_1 \le X(t) < a_2\} \tag{16}$$

Common to both triggering conditions is that the RD functions become proportional to the correlation functions only. By inserting the triggering levels in eqs. (10) - (14) the approximate prediction of the variance of the RD functions can be extracted.

3 Variance Prediction

This section introduces the theory behind a new method to predict the variance of RD functions, taking the correlation between the time segments into account. To be general the applied general triggering condition is considered. The RD functions are estimated as shown in eq. (4). The applied general triggering condition is reformulated to be a sum of conditions of the particular initial conditions at the centre of the time segements

$$\hat{D}_{YX}(\tau) = \frac{1}{N} \sum_{i=1}^{N} y(t_i + \tau)|T_{x(t_i)}^{G_A}$$

$$= \frac{1}{N} \sum_{i=1}^{N} y(t_i + \tau)|x(t_i) = x_i, \dot{x}(t_i) = \dot{x}_i$$

$$= \frac{1}{N} \sum_{i=1}^{N} y(t_i + \tau|T_{x_i}) \tag{17}$$

The N conditions, T_{x_i}, are never known beforehand but as soon the RD functions are estimated the conditions can be established. The resulting RD functions are exactly the same as the RD function estimated using the *true* triggering condition, since the time segments in the averaging process are the same. The variance of $\hat{D}_{YX}(\tau)$ is calculated from the applied general triggering condition.

$$\text{Var}[\hat{D}_{YX}(\tau)] = \frac{1}{N^2} \text{Var}[\sum_{i=1}^{N} y(t_i + \tau)|T_{x(t_i)}^{G_A}] =$$

$$\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \text{Cov}[y(t_i + \tau)|T_{x(t_i)}^{G_A}; y(t_j + \tau)|T_{x(t_j)}^{G_A}] \tag{18}$$

The applied general triggering condition is substituted by the corresponding sum of the particularly formulated triggering conditions

$$\text{Var}[\hat{D}_{YX}(\tau)] = \frac{1}{N^2} \text{Var}[\sum_{i=1}^{N} y(t_i + \tau)|T_{x_i}] =$$

$$\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \text{Cov}[y(t_i + \tau)|T_{x_i}; y(t_j + \tau)|T_{x_j}] \tag{19}$$

The double summation in eq. (19) is reformulated by applying the actual time difference known from the estimation of the RD function.

$$\text{Var}[\hat{D}_{YX}(\tau)] = \tag{20}$$

$$\frac{1}{N^2} \cdot \left(\sum_{i=1}^{N} \text{Cov}[y(t_i + \tau)|T_{x_i}; y(t_i + \tau)|T_{x_i}] \right.$$

$$+ \sum_{j=1}^{m} \sum_{i=1}^{N_j} \text{Cov}[y(t_i + \tau)|T_{x_i}; y(t_i + j\Delta T + \tau)|T_{x_{i+j}}]$$

$$\left. + \sum_{j=1}^{m} \sum_{i=1}^{N_j} \text{Cov}[y(t_i + j\Delta T + \tau)|T_{x_{i+j}}; y(t_i + \tau)|T_{x_i}] \right)$$

where m is the maximum number of time lags between any triggering points. In eq. (20) some of the N_i can be zero. The general requirement for the number of the covariance terms at each time step is

$$N + 2 \cdot N_1 + 2 \cdot N_2 + ... + 2 \cdot N_m = N^2 \tag{21}$$

Since the covariance of the conditional processes is independent of the chosen initial conditions, T_{x_i}, which will be shown later, eq. (20) can be rewritten as

$$\text{Var}[\hat{D}_{YX}(\tau)] = \tag{22}$$

$$\frac{1}{N^2} \cdot \left(\sum_{i=1}^{N} \mathrm{Cov}[Y(t+\tau)|T_{X(t)}^{G_T}; Y(t+\tau)|T_{X(t)}^{G_T}] \right.$$

$$\sum_{j=1}^{m} N_j \mathrm{Cov}[Y(t+\tau)|T_{X(t)}^{G_T}; Y(t+j\Delta T+\tau)|T_{X(t+j\Delta T)}^{G_T}] +$$

$$\left. \sum_{j=1}^{m} N_j \mathrm{Cov}[Y(t+j\Delta T+\tau)|T_{X(t+j\Delta T)}^{G_T}; Y(t+\tau)|T_{X(t)}^{G_T}] \right)$$

where $T_{X(t)}^{G_T}$ is of the same form as the theoretical general triggering condition.

$$T_{X(t)}^{G_T} = \{X(t) = a, \dot{X}(t) = b\} \tag{23}$$

The major problem is to calculate the general covariance between $Y(t+\tau)|T_{X(t)}^{G_T}$ and $Y(t+j\Delta T+\tau)|T_{X(t+j\Delta T)}^{G_T}$. Consider the following two Gaussian distributed stochastic vectors

$$\mathbf{X}_1 = [Y(t+\tau)\ Y(t+t_1+\tau)]^T \tag{24}$$

$$\mathbf{X}_2 = [X(t)\ X(t+t_1)\ \dot{X}(t)\ \dot{X}(t+t_1)]^T \tag{25}$$

The covariance of $\mathbf{X}_1$ on condition of $\mathbf{X}_2$ is calculated using the standard relation for the covariance function of conditional Gaussian distributed variables

$$\mathrm{Cov}[\mathbf{X}_1|\mathbf{X}_2] = \mathbf{R}_{\mathbf{X}_1\mathbf{X}_1} - \mathbf{R}_{\mathbf{X}_1\mathbf{X}_2}\mathbf{R}_{\mathbf{X}_2\mathbf{X}_2}^{-1}\mathbf{R}_{\mathbf{X}_1\mathbf{X}_2}^{T} \tag{26}$$

The correlation matrices in eq. (26) can be calculated from eqs. (27) - (29) ($\mathbf{X}$ and $\mathbf{Y}$ are assumed to have zero mean value)

$$\mathbf{R}_{\mathbf{X}_1\mathbf{X}_1} = \begin{bmatrix} R_{YY}(0) & R_{YY}(-t_1) \\ R_{YY}(t_1) & R_{YY}(0) \end{bmatrix} \tag{27}$$

$$\mathbf{R}_{\mathbf{X}_2\mathbf{X}_2} = \tag{28}$$

$$\begin{bmatrix} R_{XX}(0) & R_{XX}(-t_1) & -R'_{XX}(0) & -R'_{XX}(t_1) \\ R_{XX}(t_1) & R_{XX}(0) & -R'_{XX}(t_1) & -R'_{XX}(0) \end{bmatrix}$$

$$\mathbf{R}_{\mathbf{X}_1\mathbf{X}_2} =$$

$$\begin{bmatrix} R_{YX}(\tau) & R_{YX}(\tau - t_1) \\ R_{YX}(\tau + t_1) & R_{YX}(\tau) \\ R'_{YX}(\tau) & R_{YX}(\tau - t_1) \\ R'_{YX}(\tau + t_1) & R_{YX}(\tau) \end{bmatrix}$$

$$\begin{bmatrix} -R'_{YX}(\tau) & -R'_{YX}(\tau - t_1) \\ -R'_{YX}(\tau + t_1) & -R'_{YX}(\tau) \\ -R''_{YX}(\tau) & -R''_{YX}(\tau - t_1) \\ -R''_{YX}(\tau + t_1) & -R''_{YX}(\tau) \end{bmatrix}$$

The covariance between the $Y(t+\tau)|T_{X(t)}^{G_T}$ and $Y(t+n\Delta T+\tau)|T_{X(t+n\Delta T)}^{G_T}$, can be calculated by inserting the results of eqs. (27) - (29) in eq. (26). The covariance is taken as the element [1,2] of the 4×4 dimensional covariance matrix $\mathrm{Cov}[\mathbf{X}_1|\mathbf{X}_2]$.

It is important that the only information which should be available is $R_{YX}(\tau)$, $R'_{YX}(\tau)$ and $R''_{YX}(\tau)$. Since the estimated RD functions are proportional to the correlation functions the information can be obtained by scaling the RD functions and then calculate the time derivative and double time derivative of the correlation functions using numerical differentiation. This is considered to be a simple and computationally fast requirement. The disadvantage is that numerical differentiation of the correlation functions demands that the measurements are oversampled. Otherwise the terms $R'_{YX}(\tau)$ and $R''_{YX}(\tau)$ should be obtained by differentiating the measurements and then estimating the corresponding correlation functions using the RD technique. This might be the best solution if the system is not sufficiently oversampled for numerical differentiation of the RD functions.

What now remains is somehow to make the number of the different correlation functions, $N_1, N_2, \dots, N_m$, available. Instead of making some theoretical consideration of the distribution of the triggering points it is decided to use the *sample* distribution. This means that the weighting numbers, $N_1, N_2, \dots, N_m$, are obtained by picking out the time for each triggering point in the estimation of the RD functions. By sorting the time differences between the triggering points the weighting numbers are obtained.

The estimate of the variance of the RD functions involves the following computational steps.

- Sampling the time for each triggering point in estimation of the RD functions.

- Sorting the time differences between the triggering points.

- Numerical (two-time) differentiation of the RD functions (scaled to be the correlation functions).

- Calculating the variance estimate according to eq. (22).

None of these computational steps are extremely time consuming. The sampling of the time points for each

triggering point is free, since these time points are identified in the estimation process of the RD functions. In the following sections the accuracy of the method for estimating the variance of RD functions is investigated by different simulation studies.

4 Simulation Study

Consider an SDOF system loaded by Gaussian white noise. The system has an eigenfrequency of $f = 1$ Hz and a damping ratio of $\zeta = 0.05$. The response is sampled with $\Delta T = \frac{1}{30f}$ at 5000 time points. 30000 simulations of the response scaled to have unit standard deviation are performed. For each simulated response an RD function with 601 points corresponding to $-10/f \leq \tau \leq 10/f$ is calculated using the level crossing triggering condition. The time points for each triggering point are picked out and the distribution of the time points is obtained by sorting the time differences. The average number of triggering points is 185. Figure 1 shows the average simulated distribution of triggering points together with the auto correlation function.
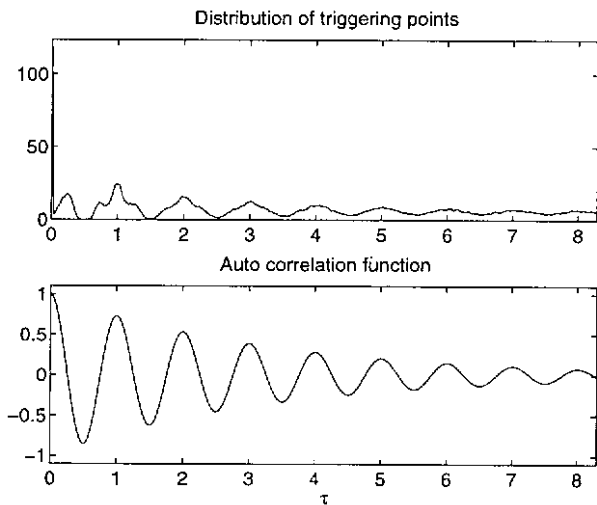
Figure 1: *Average distribution of triggering points using level crossing triggering obtained from simulations and the theoretical auto correlation function of the system.*

It is not correct to assume that the time segments in the averaging process are uncorrelated, since many triggering points are within the correlation length, τ_{max}. The correlation length is defined as $|R_{XX}(\tau) \leq \delta|$, where δ is a small number, say e.g. 0.1. Based on the 30000 independently estimated RD functions the variance of the RD function can be calculated. Figure 2 shows the simulated variance and the variance predicted by the new method where the theoretical correlation functions and the simulated distribution of triggering points have been used.
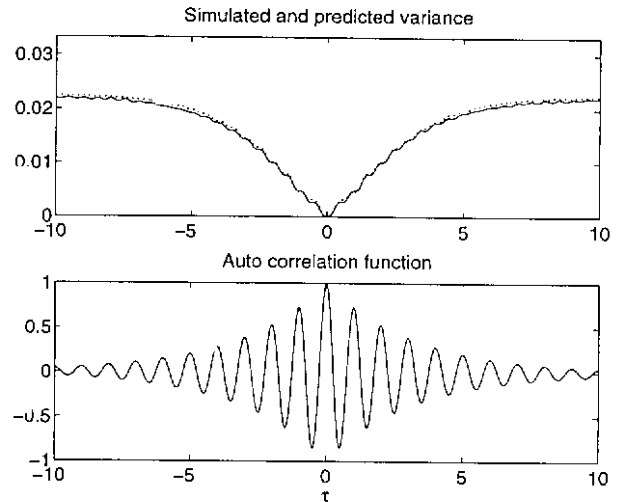

Figure 2: *The simulated and the predicted variance of the RD functions using level crossing triggering $a = 2^{0.5}\sigma_X$ and the auto correlation function of the system. [——]: Theoretical (simulated) variance of the RD function. [· · · ·]: Predicted variance of the RD function using the new method.*

As seen the new method predicts the variance of the estimated RD functions extremely well. Instead of using the simulated distribution of triggering points a single realization of the distribution of triggering points is used and instead of the theoretical auto correlation function the estimated auto correlation function is used, see fig. 3.
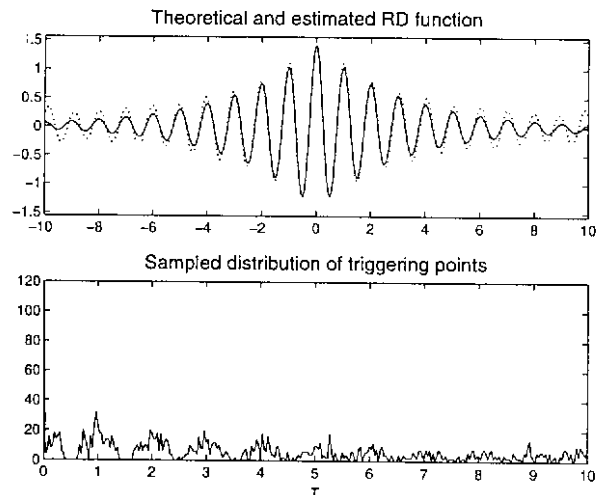

Figure 3: *The theoretical and the estimated RD function using level crossing triggering $a = 2^{0.5}\sigma_X$ and the sample distribution of the triggering points. [——]: Theoretical RD function. [· · · ·]: Estimated RD function.*

The variance predicted by the new method and the variance predicted by eq. (9) are shown together with the simulated variance in fig. 4.
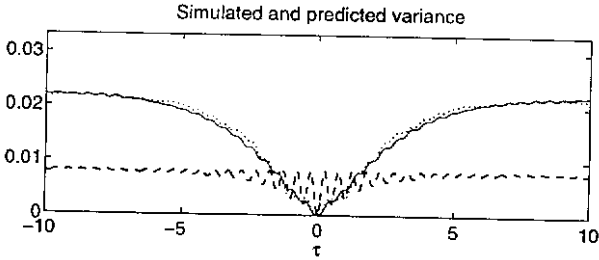
Simulated and predicted variance

Figure 4: *Simulated and predicted variance of the RD functions using level crossing triggering $a = 2^{0.5}\sigma_X$. [——]: Theoretical (simulated) variance of the RD function. [· · · ·]: Predicted variance of the RD function using the new method. [- - - -]: Predicted variance assuming uncorrelated time segments.*

As seen the new method is superior to the method where the correlation between time segments is neglected.

The positive point triggering condition is considered and applied to the same simulated responses. Figure 5 shows the simulated distribution of triggering points and the auto correlation function.
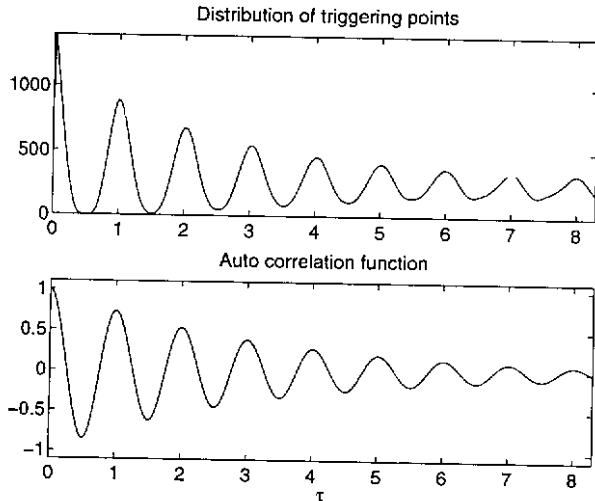
Distribution of triggering points

Auto correlation function

Figure 5: *Average distribution of triggering points obtained by simulation using positive point triggering $[a_1 \; a_2] = [\sigma_X \; \infty]$ and the autocorrelation function of the system.*

As seen it would be highly erroneous to assume that the time segments are uncorrelated. Figure 6 shows the simulated and predicted variance obtained from the theoretical auto correlation function and the simulated distribution of triggering points.
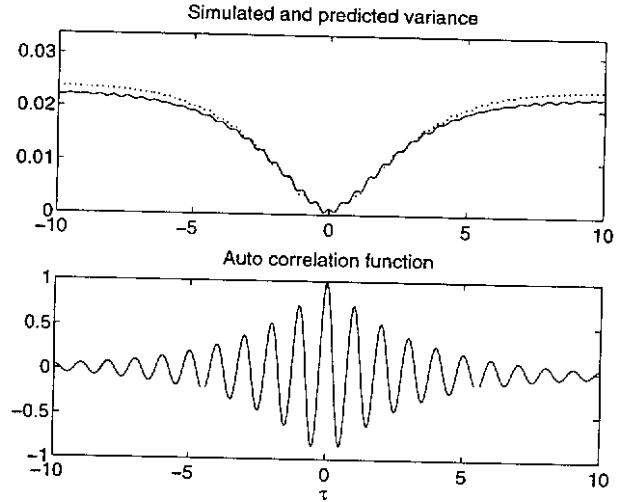
Simulated and predicted variance

Auto correlation function

Figure 6: *The simulated and the predicted variance of the RD functions using point triggering $[a_1 \; a_2] = [\sigma_X \; \infty]$ and the auto correlation function of the system. [——]: Theoretical (simulated) variance of the RD function. [· · · ·]: Predicted variance of the RD function using the new method.*

Figure 7 shows the estimated RD function, the theoretical RD function and the distribution of triggering points for a single realization of the response.
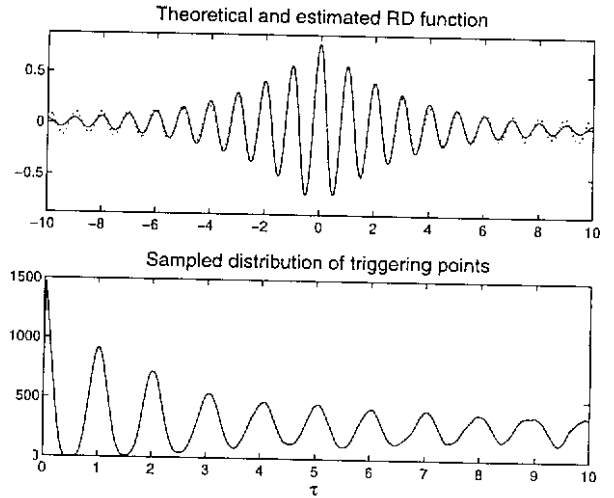
Theoretical and estimated RD function

Sampled distribution of triggering points

Figure 7: *The theoretical and the estimated RD function using positive point triggering $[a_1 \; a_2] = [\sigma_X \; \infty]$ and the sample distribution of the triggering points. [——]: Theoretical RD function. [· · · ·]: Estimated RD function.*

The variance predicted using eq. (9) and the variance predicted by the new method are shown in fig. 8, where the estimated RD function and the distribution of triggering points from a single realization are used.
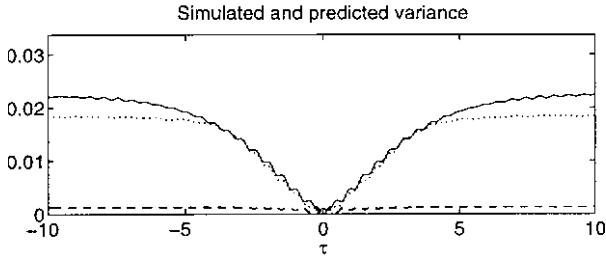
Figure 8: *The simulated and the predicted variance of the RD functions using positive point triggering $[a_1\ a_2] = [\sigma_X\ \infty]$. [——]: Theoretical (simulated) variance of the RD function. [·····]: Predicted variance of the RD function using the new method. [- - - -]: Predicted variance assuming uncorrelated time segments.*

The new method is superior to the method based on eq. (9) and predicts the variance very well.

In order to look at a more complicated system and cross RD functions a 2DOF system with the following modal parameters is considered.

f [Hz]	ζ [%]	$|\Phi|^1$	$|\Phi|^2$	$\angle\Phi^1$	$\angle\Phi^2$
3.74	4.10	1.000	1.005	0.00	177.7
6.27	4.50	1.000	0.995	0.00	1.3

Table 1: *Modal parameters of a 2DOF system.*

The theoretical correlation (RD) functions of the system are shown in fig. 9.
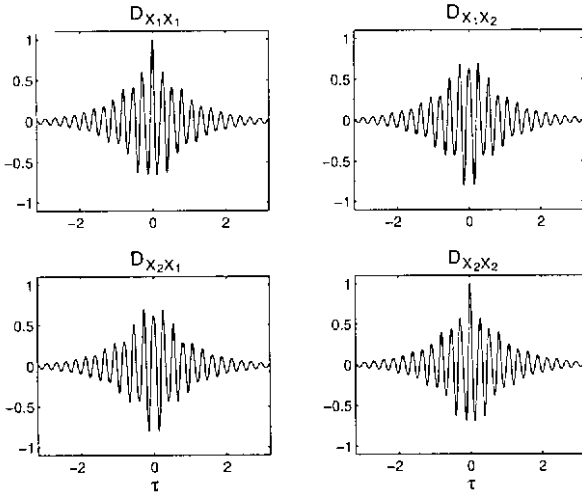

Figure 9: *Theoretical correlation functions of a 2DOF system.*

The investigations are based on 50000 simulations of the response of the system loaded by white noise followed up by an estimation of the RD functions using the positive point triggering condition with the triggering levels

$[a_1\ a_2] = [\sigma_X\ \infty]$. Figure 10 shows the simulated distribution of the triggering points and a single realization of the distribution of triggering points.
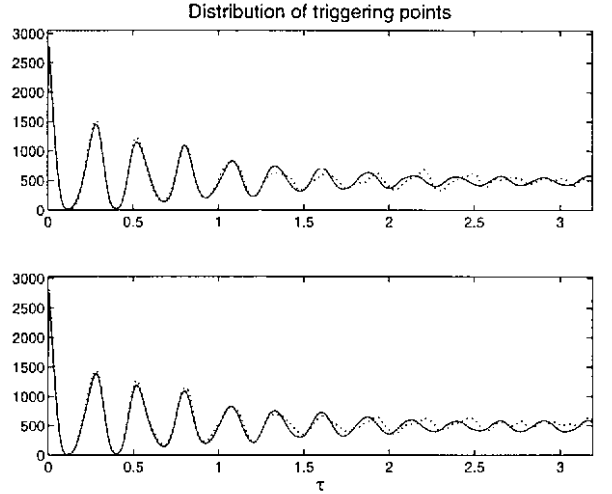

Figure 10: *[——]: Simulated distribution of triggering points. [·····]: Distribution of triggering points from a single realization.*

The distribution of triggering points is well described by a single realization of the measurements. Figures 11 and 12 show the simulated variance of the RD functions, the variance predicted by eq. (9), the variance predicted by the new method using the theoretical RD functions and simulated distribution of triggering points and the variance predicted by the new method using a single realization of the distribution of triggering points and the correspondingly estimated RD function.
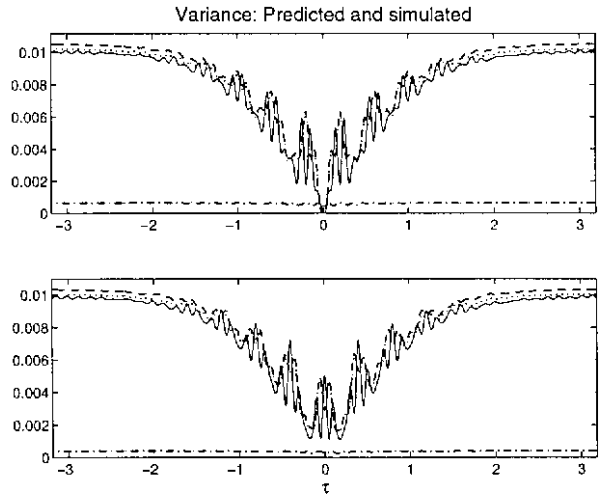

Figure 11: *Variance of RD functions. [——]: Simulated variance. [- - - -]: Variance predicted from a single realization. [·····]: Variance predicted from theoretical RD function and simulated distribution of triggering points. [— · — ·—]: Variance predicted by eq. (3.46).*

Figure 12: *Variance of RD functions.* [———]: *Simulated variance.* [- - - -]: *Variance predicted from a single realization.* [· · · ·]: *Variance predicted from theoretical RD function and simulated distribution of triggering points.* [— · — · —]: *Variance predicted by eq. (3.46).*
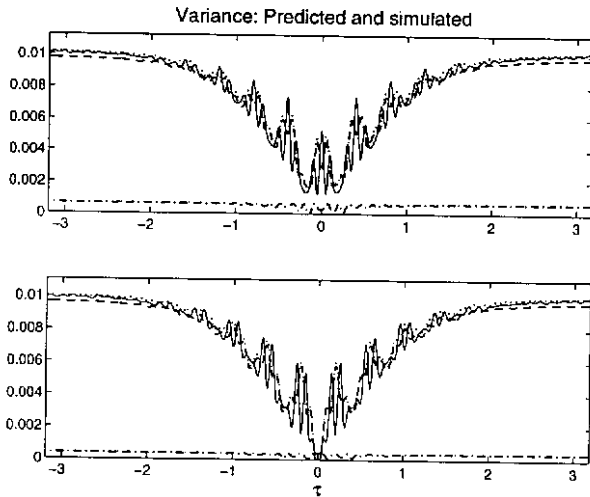
The investigation of this 2DOF system also show that the new method is superior to the predictions by eq. (9).

5 Conclusions

An approach to predict the variance of RD functions has been suggested. The method takes the correlation between the time segments into account by using the sampled time points of the triggering points. The method has been tested by simulation of different systems. The method seems to predict the variance very well at the zero time lag and for time lags where the variance has converged. It is superior to the method for predicting the variance, which is based on uncorrelated time segments in the averaging process. It is an open question if this increase in accuracy can pay off the increasing computational time. Further investigations in order to understand the approach are recommended.

6 Acknowledgement

References

[1] Cole, H.A. *On-The-Line Analysis of Random Vibrations.* AIAA Paper No. 68-288. 1968.

[2] Cole, H.A. *Method and Apparatus for Measuring the Damping Characteristics of a Structure.* United States Patent No. 3, 620,069, Nov. 16. 1971.

[3] Cole, H.A. *Failure Detection of a Space Shuttle Wing by Random Decrement.* NASA TMX-62,041, May 1971.

[4] Cole, H.A. *On-Line Failure Detection and Damping Measurements of Aerospace Structures By Random Decrement Signature.* NASA CR-2205, 1973.

[5] Ibrahim, S.R. *Random Decrement Technique for Modal Identification of Structures.* Journal of Spacecraft and Rockets, Vol. 14, No. 11., Nov. 1977, pp. 696-700.

[6] Ibrahim, S.R. *The Use of Random Decrement Technique for Identification of Structural Modes of Vibration.* AIAA paper, Vol. 77, 1977, pp. 1-9.

[7] Vandiver, J.K., Dunwoody, A.B., Campbell, R.B. & Cook, M.F. *A Mathematical Basis for the Random Decrement Vibration Signature Analysis Technique.* Journal of Mechanical Design, Vol. 104, April 1982, pp. 307-313.

[8] Brincker, Krenk, S. & R., Jensen, J.L. *Estimation of Correlation Functions by the Random Dec Technique.* Proc. Skandinavisk Forum for Stokastisk Mekanik, Lund, Sweden, Aug. 30-31, 1990.

[9] Brincker, R., Krenk, S. & Jensen, J.L. *Estimation of Correlation Functions by the Random Decrement Technique.* Proc. 9th International Modal Analysis Conference and Exhibit, Firenze, Italy, April 14-18, 1991.

[10] Brincker, R., Krenk, S., Kirkegaard, P.H. & Rytter, A. *Identification of Dynamical Properties from Correlation Function Estimates.* Bygningsstatiske Meddelelser, Vol. 63, No. 1, 1992, pp. 1-38.

[11] Asmussen, J.C. *Modal Analysis Based on the Random Decrement Technique - Applications to Civil Enginering Structures.* Ph.D.-Thesis Aalborg University, Denmark, August 1997.