Scaling the Mode Shapes of a Building Model by Mass Changes

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NOMENCLATURE

$\boldsymbol{\varphi}$	Mode shape vector scaled to unity
$\boldsymbol{\psi}$	Mode shape vector scaled to normalize with mass matrix
ω	Natural frequency
$\Delta\omega$	Natural frequency shift
α	Scaling factor
$\mathbf{M}$	Mass matrix
$\Delta\mathbf{M}$	Mass change matrix
Δm	Scalar mass change
$\mathbf{D}$	Diagonal matrix containing only unity values
$\Delta\mathbf{m}$	Mass change vector
σ	Standard deviation

ABSTRACT

It is well known, that when using natural input modal analysis, the loads are not known, and thus, the mode scaling factor that relates the magnitude of the loading to the magnitude of the response cannot be estimated. However It has been pointed out by several theoretical papers that mode shapes can be scaled by performing several natural input modal analysis tests with different mass changes, observe the frequency shift introduced by the mass changes and then follow an estimation scheme that allows the user to estimate the scaling factor mode-by-mode, i.e. only information of the particular mode of interest is used to obtain the scaling factor for that mode. The procedure is of high practical interest also in mechanical engineering since it is well known that the traditionally estimated scaling factor is often suffering from large uncertainties. In this paper it is shown how the mass change technique can be used on a ¼ scale model of a 4-storey building. The uncertainties on the estimated scaling factors are illustrated by repeating the estimation using different mass changes.

INTRODUCTION

Output-only modal testing and analysis – or as it may be should be called: Natural input modal testing and analysis - is becoming more and more popular due to the clear advantages of the technology: The testing is easier, the technology is applicable to a wider a range of the structures and it has a wider range of potential applications since the actual responses are stored and can be used for instance fatigue analysis and vibration level estimation. Also the technology gives better and more reliable results in cases where the actual loading conditions and operating conditions are important for the structural response.

However the technology is still in the early stages of development and many problems still remain unsolved or partly unsolved. One of the important remaining problems is the problem of mode shape scaling. If the identified modal model is going to be used for structural response simulation or for structural modification, then the scaling factors of the mode shapes must be known. Also in health monitoring applications and in cases where damage is to be identified, the scaling factors might be useful.

Recently some suggestion has been given in the literature for solving this problem. One solution has been suggested by Bernal and Gunes [1] based on the assumption that partition of the inverse of the mass matrix associated with the measured coordinates is diagonal. However, the approach gives exact answers only when there is a full set of modes, and robustness for a truncated modal space has not been demonstrated. Recently Parloo *et al.* [2] have published a new approach based on a more extensive testing procedure that involves repeated testing where mass changes are introduced in the points of the structure where the mode shape is known. This approach seems more appealing, since to scale a certain mode, only that particular mode has to be known.

Brincker *et al.* [3], [4] have published an improved procedure following the ideas of Parloo *et al.* and also performed an uncertainty analysis that clearly indicates a lower uncertainty on the mode shape when following this estimation procedure for the scaling factor than what is know to be the case using traditional modal analysis.

In Brincker *et al.* [3], [4] it is shown that if one arbitrary mode shape $\boldsymbol{\varphi}$ estimated by natural input modal analysis is scaled to unity, i.e. so that $\boldsymbol{\varphi}^{\mathbf{T}}\boldsymbol{\varphi}=1$ and the corresponding mode shape $\boldsymbol{\psi}$ scaled as usual so that $\boldsymbol{\psi}^{\mathbf{T}}\mathbf{M}\boldsymbol{\psi}=1$, then the scaling factor α relating the two mode shape estimates $\boldsymbol{\psi}=\alpha\,\boldsymbol{\varphi}$ can be found by introducing a mass change and obtaining the estimate as

$$\alpha=\sqrt{\frac{\omega_1^2-\omega_2^2}{\omega_2^2\,\boldsymbol{\varphi}^{\mathbf{T}}\boldsymbol{\Delta}\mathbf{M}\,\boldsymbol{\varphi}}} \tag{1}$$

where $\boldsymbol{\Delta}\mathbf{M}$ is the mass change matrix and ω_1, ω_2 are the natural frequency of the considered mode before and after application of the mass change. If the mass change matrix is proportional to the initial mass matrix, then the mode shapes will not change, and equation (1) is then an exact linear elastic solution for the relationship between mass changes, frequency shift and mode shape scaling factor. In the case where the mass change matrix is not proportional to the initial mass matrix, equation (1) becomes approximate.

Assuming that the uncertainty σ_α on α is controlled by the uncertainty $\sigma_{\Delta\omega}$ on the frequency shift $\Delta\omega$ and by the uncertainty σ_φ on the mode shape coordinates, then, if the mass change matrix can be written $\boldsymbol{\Delta}\mathbf{M}=\Delta m\mathbf{D}$, where the matrix $\mathbf{D}$ is a diagonal matrix with only unity values and zeroes in the diagonal, then the uncertainty on the scaling factor can be estimated from

$$\frac{\sigma_\alpha^2}{\alpha^2}=\frac{1}{4}\frac{\sigma_{\Delta\omega}^2}{\Delta\omega^2}+\frac{\sigma_\varphi^2}{\boldsymbol{\varphi}^{\mathbf{T}}\mathbf{D}\,\boldsymbol{\varphi}} \tag{2}$$

This formula indicates, that if $\Delta\omega=\omega_1-\omega_2$ is 10 % of the initial natural frequency, if the accuracy on $\Delta\omega$ is 1 % of the initial frequency, and if we place masses in approximately 8 out of 36 of the measurement points (as we can say approximately that we are doing for the structure under consideration), then the uncertainty contribution due to frequency is approximately 2.5 % and the uncertainty contribution due to mode shape uncertainty is approximately 0.5 %, thus the total uncertainty due to statistical errors can be estimated to be of the order of 3 %. However, performing tests on real structures, errors from nonlinearities, from uncertain estimation of the mass change matrix, and from mode shape changes must expect significantly to increase the uncertainty on the mode shape factor.

The main purpose of the investigation described in this paper is to document uncertainties on the mode shape factor from a realistic test – a test that in its testing conditions is close to a real in-situ civil engineering testing situation.

NATURAL INPUT TESTING

The structure that was tested is shown in Figure 1. It is a four storey building model in scale 1:4, cast in concrete, slab thickness 50 mm, 750 mm between floors, each deck 1250x2250 mm. The structure was partly damaged during a shaking table test before testing, thus opening and closure of cracks must expect to contribute to structural non-linearities as the mass is increased.

The structure was tested in four different situations:

- Test A, structure is mass loaded by one layer of loading blocks (initial state)
- Test B, some additional loading blocks added in a second layer, 64 kg on each floor
- Test C, some additional loading blocks added in a second layer, 128 kg on each floor
- Test D, some additional loading blocks added in a second layer, 256 kg on each floor

Total mass of the structure included the initial layer of loading blocks was estimated to be 4153 kg.

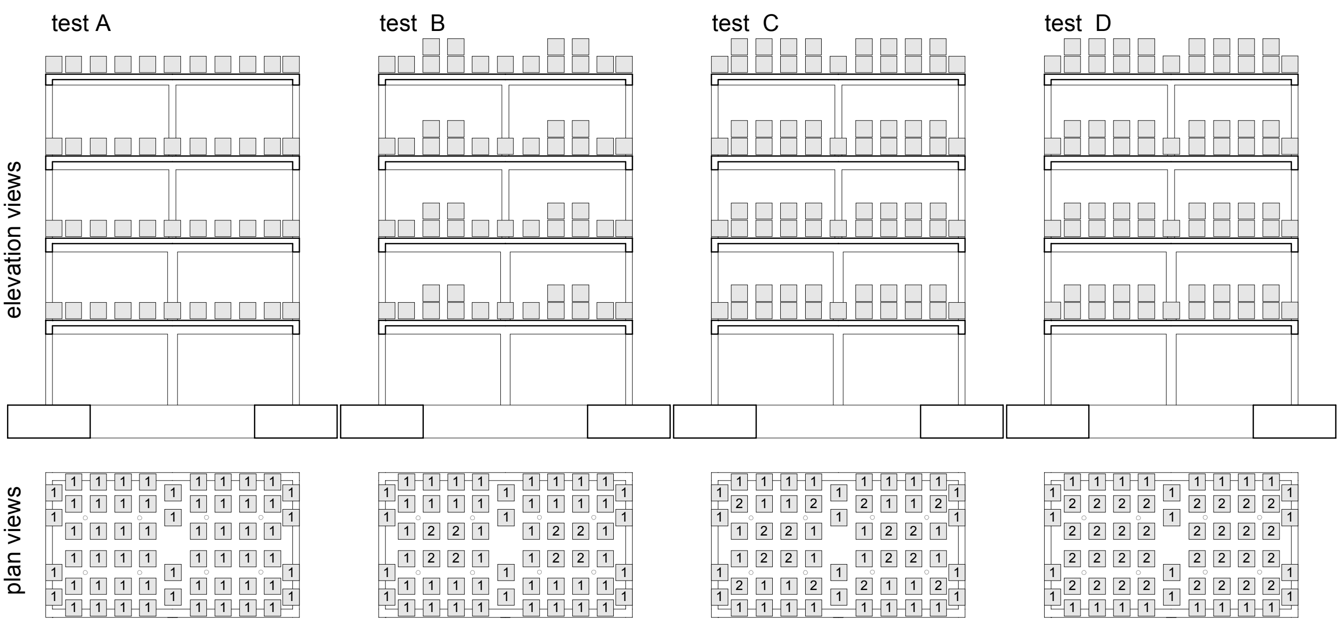

Figure 1. *The test structure with the loading blocks on each floor in each of the test situations.*

The structure was loaded only by natural excitations from what was expected to be mainly micro vibrations from the ground due to traffic and small movements of the air due to ventilation through the laboratory from open doors. The response was measured using 12 uni-axial EPI sensors from Kinemetrics configured as shown in Figure 2. Signals were sampled using a 16 bit acquisition system with a resolution (LSB) of 0.076 μg and a sensitivity of 4 Volts/mg. The tests were carried out with a sampling frequency of 1000 Hz, and later the time series were low-pass filtered using a 8 poles Butterworth digital filter with a cut-off frequency of 25 Hz and thereafter decimated to a sampling frequency of 62.5 Hz corresponding to a Nyquist frequency of 31.25 Hz. In each test the natural responses were measured during a period of approximately 1 hour corresponding to approximately 230,000 data points in each time series. The test was performed at LNEC, Lisbon. More information about the test structure can be found in Rodrigues *et al.* [5] and Coelho *et al.* [6].

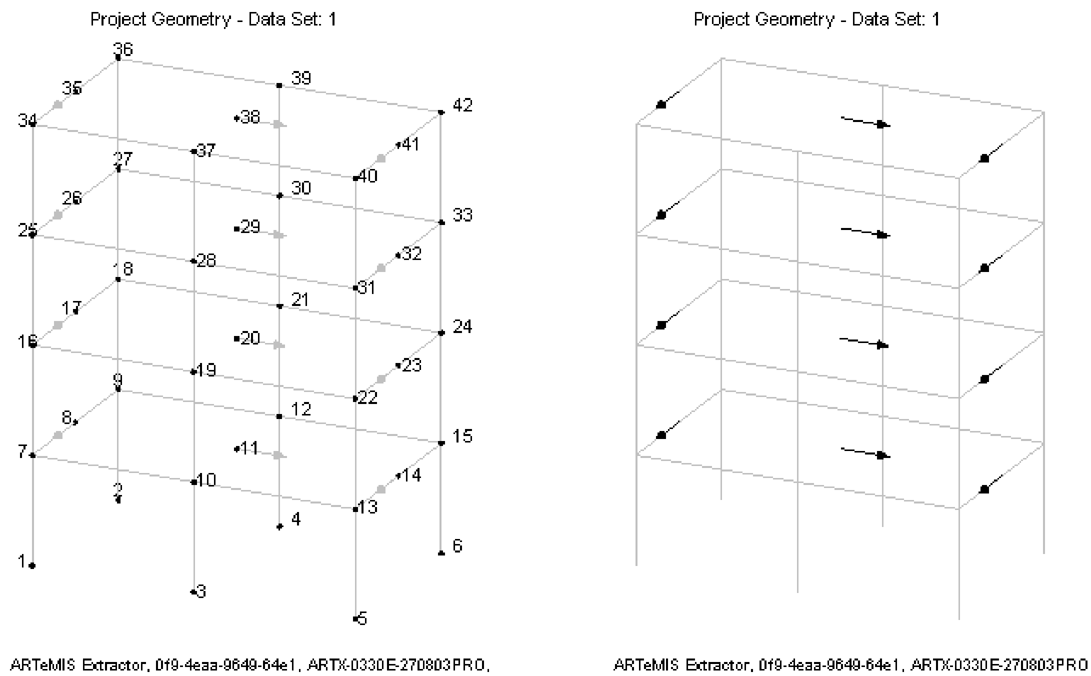

Figure 2. *Node points and measurement point layout*

MODAL IDENTIFICATION

All modes were identified using the Frequency Domain Decomposition (FDD) technique, Brincker *et al.* [7], and the Stochastic Subspace Identification (SSI) technique, Overschee and De Moor [8], using the ARTeMIS Extractor software. 11 modes in the frequency range 3 – 22 Hz could be identified, see Figures 3 and 4. However, comparing the modes estimated by FDD for case A (initial state) with case D (the case with the hardest mass loading) reveals that only the first six modes are reasonably unchanged from case A – D, see the MAC matrix in Figure 5. Thus, only the first six modes are included in the further analysis. The first six modes are the two first bending modes in the longitudinal and the transverse direction of the building and the first two rotational modes.

The estimated natural frequencies by FDD and SSI for the first six modes are shown in Table 1 and Table 2.

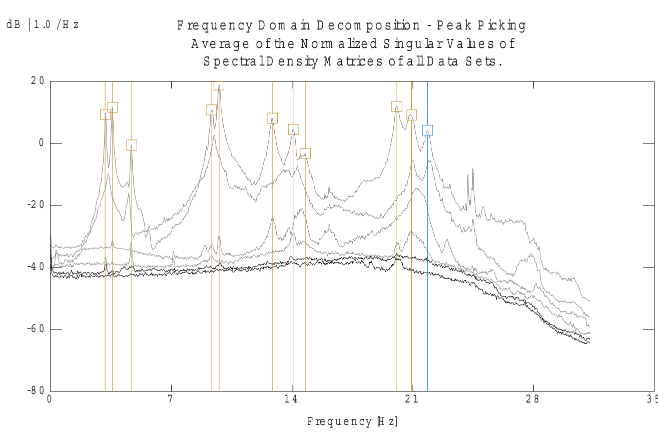

Figure 3. *Modes identified for case A by the FDD technique*

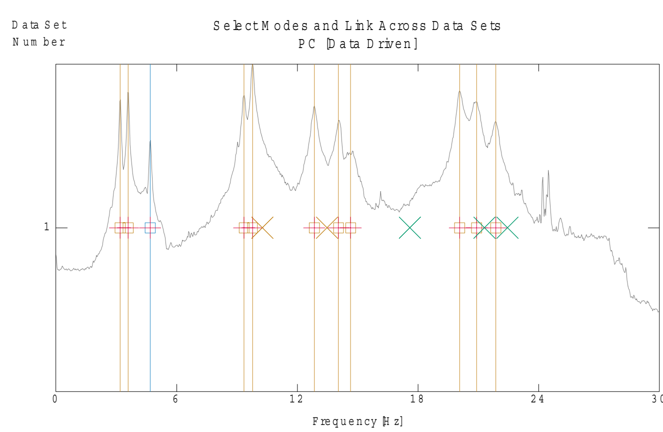

Figure 4. *Modes identified for case A by the SSI technique*

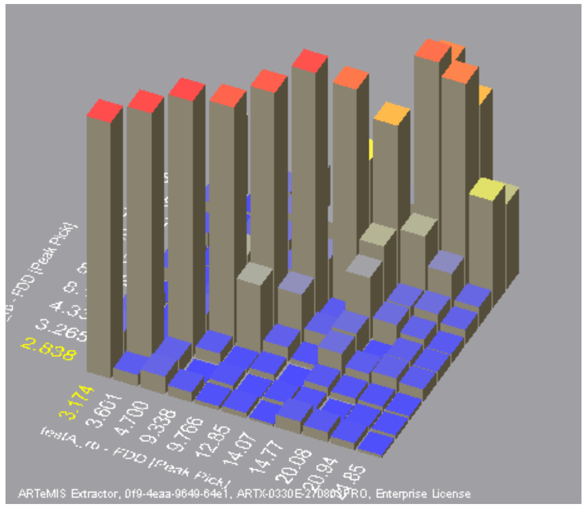

Figure 5. *MAC matrix between modes estimated by FDD for case A (initial state) and case D (hardest mass loading)*

Table 1. *Natural frequencies estimated by FDD*

Mode	Frequency [Hz] case A	Frequency [Hz] case B	Frequency [Hz] case C	Frequency [Hz] case D
Mode 1	3.174	3.082	2.991	2.838
Mode 2	3.601	3.479	3.387	3.265
Mode 3	4.700	4.608	4.517	4.333
Mode 4	9.338	9.155	8.789	8.148
Mode 5	9.766	9.491	9.216	8.972
Mode 6	12.85	12.54	12.24	11.54

Table 2. *Natural frequencies estimated by SSI*

Mode	Frequency [Hz] case A	Frequency [Hz] case B	Frequency [Hz] case C	Frequency [Hz] case D
Mode 1	3.191	3.086	2.996	2.829
Mode 2	3.588	3.483	3.391	3.258
Mode 3	4.686	4.584	4.462	4.400
Mode 4	9.354	9.154	8.879	8.174
Mode 5	9.770	9.515	9.227	9.046
Mode 6	12.84	12.53	12.30	11.55

MODE SHAPE SCALING

The above referred procedure for estimating the mode shape scaling factor assumes that masses are added only in the points where the mode shape is known. In the present case – as it appears from Figure 1 – the applied added masses are distributed on the decks of the building in a way so that it is not clear how the mass changes should be distributed on the node points.

In this analysis the mass changes are distributed in the simplest possible way so that the mass distribution is symmetrical and gives the same moment of inertia around the two horizontal principal axes for the deck plates. For each deck plate nine node points has been defined, see Figure 2 and the corresponding mass distribution is defined in such a way that for instance the node numbers 7,8,…15 for the first deck is given as $n_k, k = 1,2...9$ and the mass change of node n_k is given as the k'th element $\Delta\mathbf{m}(k)$ of the mass change vector $\Delta\mathbf{m}$ for the deck.

Thus, omitting the first six node points (the base nodes) from the analysis, the mass change matrix for the whole structure can be defined as

$$\Delta \mathbf{M} = \begin{bmatrix} diag(\Delta \mathbf{m}) & 0 & 0 & 0 \\ 0 & diag(\Delta \mathbf{m}) & 0 & 0 \\ 0 & 0 & diag(\Delta \mathbf{m}) & 0 \\ 0 & 0 & 0 & diag(\Delta \mathbf{m}) \end{bmatrix} \tag{3}$$

which is then a 36x36 diagonal matrix. The inner product $Z = \boldsymbol{\varphi}^{\mathbf{T}} \Delta \mathbf{M} \boldsymbol{\varphi}$ is in practice carried out direction-by-direction as $Z = \boldsymbol{\varphi}_x^T \Delta \mathbf{M} \boldsymbol{\varphi}_x + \boldsymbol{\varphi}_y^T \Delta \mathbf{M} \boldsymbol{\varphi}_y + \boldsymbol{\varphi}_z^T \Delta \mathbf{M} \boldsymbol{\varphi}_z$ where indices indicate the different coordinate directions. The three different loading cases can be combined as six different changes, namely changes between cases A-B, A-C, A-D, B-C, B-D and finally C-D. In Tables 4 and 5 is given the mode shape scaling factor estimated using equation (1) for all combinations and for the first six modes of the structure. As it appears from the results, the empirical standard deviation is typically in the range 10-15 % and there does not seem to be any quality difference between FDD and SSI estimates. Since SSI identification is known to be significantly more accurate in estimating natural frequencies than the simple peak picking FDD, this indicates that the observed uncertainty is not significantly influenced by natural frequency estimation uncertainty. Also it was observed that the values of the scaling factor was only weakly dependent upon what mode shape was used in equation (1) (one can use the initial mode shape or the mode shape corresponding to the applied mass loading – results in Tables 4 and 5 are the average of the two). Finally, since the rotational modes (mode 3 and mode 6) show the same uncertainty on the scaling factor as the bending modes, and since only the rotational modes are sensitive to how the mass changes are distributed over the nodes on each deck, it can be concluded, that errors on the mass change matrix does not contribute significantly to the observed uncertainty. Thus, it can be concluded that for the present test case, the uncertainty was mainly governed by other sources than simple statistical errors on natural frequencies and mode shape coordinates and errors introduced in estimating the mass change matrix. Main results for the mode shape scaling factors are given in Table 6 and as it appears, results from the two modal estimators are coherent.

Table 3. *Mass change vectors* $\Delta \mathbf{m}$ *for the three mass loading cases*

	Case B [kg]	Case C [kg]	Case D [kg]
Mass added in node 7	0	0	0
Mass added in node 8	8.28	19.2	38.4
Mass added in node 9	0	0	0
Mass added in node 10	1.44	12.8	25.6
Mass added in node 11	44.56	64	128
Mass added in node 12	1.44	12.8	25.6
Mass added in node 13	0	0	0
Mass added in node 14	8.28	19.2	38.4
Mass added in node 15	0	0	0
Total added mass on 1 floor	64	128	256

Table 4. *Scaling factors based on FDD modal estimation*

Mode	A-B $[\text{kg}^{-1/2}]$	A-C $[\text{kg}^{-1/2}]$	A-D $[\text{kg}^{-1/2}]$	B-C $[\text{kg}^{-1/2}]$	B-D $[\text{kg}^{-1/2}]$	C-D $[\text{kg}^{-1/2}]$	Mean $[\text{kg}^{-1/2}]$	Std $[\text{kg}^{-1/2}]$	std/mean [%]
1	0.0928	0.0948	0.1098	0.0939	0.1141	0.1279	0.106	0.014	13.4
2	0.1003	0.0957	0.1026	0.0878	0.1003	0.1097	0.099	0.007	7.4
3	0.1322	0.1163	0.1448	0.1041	0.1461	0.1870	0.138	0.029	20.9
4	0.0771	0.0973	0.1070	0.1112	0.1129	0.1098	0.103	0.014	13.2
5	0.0909	0.0930	0.0816	0.0923	0.0758	0.0635	0.083	0.012	14.1
6	0.1376	0.1235	0.1392	0.1122	0.1365	0.1471	0.133	0.013	9.5

Table 5. Scaling factors based on SSI modal estimation

Mode	A-B $[kg^{-1/2}]$	A-C $[kg^{-1/2}]$	A-D $[kg^{-1/2}]$	B-C $[kg^{-1/2}]$	B-D $[kg^{-1/2}]$	C-D $[kg^{-1/2}]$	Mean $[kg^{-1/2}]$	Std $[kg^{-1/2}]$	std/mean [%]
1	0.0994	0.0977	0.1144	0.0929	0.1173	0.1343	0.109	0.016	14.3
2	0.0932	0.0920	0.1020	0.0880	0.1032	0.1146	0.099	0.010	9.8
3	0.1412	0.1309	0.1224	0.1221	0.1144	0.1013	0.122	0.014	11.2
4	0.0809	0.0899	0.1064	0.0958	0.1113	0.1153	0.100	0.013	13.3
5	0.0876	0.0925	0.0779	0.0946	0.0721	0.0547	0.080	0.015	18.8
6	0.1413	0.1177	0.1393	0.0986	0.1352	0.1513	0.131	0.019	14.6

Table 6. Scaling factor main results

Mode	FDD $[kg^{-1/2}]$	SSI $[kg^{-1/2}]$
1	0.106	0.109
2	0.099	0.099
3	0.138	0.122
4	0.103	0.100
5	0.083	0.080
6	0.133	0.131

CONCLUSIONS

Scaling factors have been estimated on a four storey concrete building model in scale 1:4 by applying three cases of added masses that are approximately 6 %, 12 % and 24 % of the initial mass. Using two different estimators for natural input modal analysis, the Frequency Domain Decomposition and the Stochastic Subspace Identification technique, natural frequencies and mode shapes were estimated for the first six modes and the mode shape scaling factors were estimated for these six modes for all six mass change combinations.

The results show clearly that for the present test case, the mode shape scaling factors has been estimated with an empirical standard deviation of 10-15 %. It can also be concluded, that the statistical errors on natural frequencies and mode shape coordinates and errors from estimating the mass change matrix only contribute insignificantly to the observed uncertainty. The observed uncertainty is believed mainly to be due to nonlinearities caused by the fact that the structure was damaged in a shaking table test before performing the natural input testing.

Further investigations on sources of errors that can significantly influence the uncertainty on the scaling factor are recommended.

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