An Overview of Major Developments and Issues in Modal Identification

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ABSTRACT During a revisit to modal identification in the past three decades, an overview of the major developments and issues in modal identification is motivated. A brief review on the progress from Single-Input/Single-Output (SISO), Single-Input/Multi-Output (SIMO) to Multi-Input/Multi-Output (MIMO) identification in frequency, time and spatial domain, is presented. The latest developments and issues are then discussed, which cover (1) Traditional Experimental Modal Analysis (EMA) using both Input/Output measurements and Operational Modal Analysis using Output data only; (2) Two-stage modal identification and one-stage modal identification, and (3) Deterministic modal identification and statistical modal identification, which takes measurement noise and system uncertainty into account.

1 INTRODUCTION

Modal identification (MID) has obtained substantial progress in the last three decades. Numerous modal identification algorithms, from Single-Input/Single-Output (SISO), Single-Input/Multi-Output (SIMO) to Multi-Input/Multi-Output (MIMO) techniques in Time Domain (TD), Frequency Domain (FD) as well as Spatial Domain (SD), have been developed. Experimental modal analysis has been widely used in trouble shooting, structural dynamics modification, analytical model updating, optimal dynamic design, passive & active vibration control, as well as vibration-based structural health monitoring in aerospace, mechanical and civil engineering.

MID can be thought as a branch of general system identification for mechanical structures. System identification is defined as to build math model of a dynamic system via measured input and output data. Its major part is parameter estimation for a parametric model. System identification has been developed rapidly in 1960's, and became a major research direction and new discipline in control & system engineering in the early 1970's.

There was an interesting and important workshop on "System Identification of Vibration Structures", sponsored by the Shock & Vibration Committee of the Applied Mechanics Division, ASM, took place in 1972. A book on the same title followed in the same year, which documented the state-of-the-art of system identification as related to shock and vibration, including methodology and application [1]. Two papers dealt with the state-of-the-art of system identification in general. The rest of seven papers were focused on applications in aerospace, automobile, machine tool, as well as civil engineering.

It was interesting to observe the development of system identification for mechanical structures after the workshop. MID in mechanical engineering led a little different way and focused on frequency response function (FRF) or impulse response function (IRF) measurements. Meantime, in civil engineering the same way was followed as in system/control engineering featured in not only modal but physical parameter estimation. Even statistical view point was emphasized in the very beginning. However the main stream of MID was the methods based on FRF/IRF data under deterministic framework. MID has been developed very fast in 1970's, and early 1980's along its unique way, and successfully utilized in mechanical, aerospace as well as civil engineering.

In the middle of 1980's, a time domain MID technique, Eigensystem Realization Algorithm (ERA) was proposed based on state-space model, which was actually "re-planted" from System Realization in system/control engineering and applied for modal identification in aerospace engineering. Many variations of system realization techniques were developed followed ERA. Another direction in MID is also along the same way as general system identification based on time series model. Subspace State-space System (4SID) Identification, a better approach compared to ARMA-type model in traditional system identification region, was developed in 1990's. Meanwhile, frequency domain approaches based on transfer function using Matrix Fraction Description were also applied for MID. It seems that the directions of system identification in mechanical/structural engineering and system/control engineering have merged

MID can then be utilized not only with input and output measurements, but also with output-only data. It is possible to extract modal parameters in two stages, having FRF or IRF estimation as the first stage and modal parameter estimation as the second stage, but also accomplished in one stage by directly using measured input and output data. Modal identification is no longer built within deterministic, but also stochastic framework, depending on the noise pollution and system distortion, as well as the necessity of confidence interval for identified parameters.

This paper is to provide an overview of the major developments and issues in modal identification. The rest of the paper is arranged as follows. A brief review of the development of MID in the first two decades is briefly summarized in next section. The major development in the third decade with major issues will be discussed in the following three sections: Operational Modal Analysis (OMA), or output-only (O/O) modal identification, versus EMA with Input and Output (I/O) measurements; two-stage versus one-stage MID; statistical versus deterministic framework in MID.

2 A BRIEF REVIEW OF MID

Modal identification (MID) started from nonparametric determination of modal parameter based on different representation of measured FRFs with amplitude/phase, real/imaginary and Nyquist formats. Parametric MID was the significant advance in 1970's. Complex Exponential (CE) algorithm based on Prony's method was proposed in 1974, which can be classified as the first important SISO parametric MID method. CE algorithm has then been extended into SIMO version based on Least Squares (LS) estimation, and named as LSCE. The paper on LSCE was published two years latter [2]. In the same year, i.e. 1977, well-know Ibrahim Time Domain (ITD) method was proposed [3]. ITD was, probably the first MID algorithm formulated in SIMO version, and featured in solving eigenvalues (and then modal frequencies and damping ratios) and eigenvectors (mode shapes) in one step.

Modal identification has obtained substantial progress in its second decade, i.e. 1980's, marked with two milestones: (1) MID in Frequency Domain (FD) and (2) from SIMO to MIMO. Rational Fraction Polynomial (RFP) based on orthogonal, e.g. Forsythe, polynomials was the first major FD MID technique, developed in 1982 [4]. RFP was extended from its SISO to SIMO version in 1985 [5].

The first MIMO MID method, Polyreference Complex Exponential (PRCE [6], 1982), was a milestone in the Experimental Modal Analysis (EMA), which is actually an extension of LSCE algorithm. With MIMO MID, EMA has the capability of handling complex structures with closely-spaced or even repeated modes since then. Not only multi-column/row FRF data can be utilized separately, as references, but simultaneously, for MIMO MID. The ITD algorithm, as a SIMO MID algorithm has been extended into MIMO version as EITD [7]. Eigensystem Realization Algorithm (ERA [8], 1984), based on general state-space description of linear dynamic systems, was another breakthrough in MID. Many different versions, e.g. ERA/OKID [9], Q-Markov Covariance Equivalent Realization (QMC) [10], which can be thought of extension of System Realization from system/control engineering.

Most TD MID techniques make use of free-decay, impulse response or random decrement data. It has been realized that identification accuracy can be improved by applying “correlation filter” or data correlation to noisy time response data. An Improved PRCE and ERA/DC were then developed in 1987 [11] and 1988 [12], respectively.

In FD, the first MIMO MID method was proposed in 1985, named as Frequency Domain Polyreference (FDPR) [13]. At the same time a similar version called as FD Direct Parameter Identification was developed [14]. It should be mentioned that FDDPI is more general version for it can make use of not only measurement data with their first-order derivatives, as FDPR, but possibly plus second-order ones. SIMO RFP method was extended into its MIMO version by different authors, e.g. [15]. A paper on FD ERA was also published in 1988 [16].

A number of MID algorithms have been developed individually via different authors with different formulations. Comparison of different algorithms was of interest and importance in second decade of MID. It became possible when reformulated the algorithms in the common mathematical structure. A unifying approach was proposed for TD MID [17]. The author of this paper made a presentation at the Advanced Seminar on Modal Analysis held in the University of Cincinnati in 1985 to reveal the interrelationship between PRCE, EITD, ERA and their FD counterparts, i.e. MIMO-RFP and PRFD, as well their SIMO versions as special cases. A well-known Unified Matrix Polynomial Approach (UMPA) was proposed covering large amount of major algorithms developed in the first two decades of MID in a common framework based on multiple-dimension Auto-regression eXogenous (ARX) model. [18]. Textbooks on MID were also published in early 1990’s [19]-[21]. The theory in book [19] was based on time series model or Auto-regression Moving Average (ARMA) model (actually ARX model!) and applied to EMA. On the other side, book [20] provides a thorough discussion of TD MID based on common framework via stare-space model. Book In [21], FD MID based on transfer function model in rational fraction (or matrix) description is summarized.

MID seems matured in the late of 1980’s and early 1990’s. However, lots of issues need to be addressed. Most MID methods developed in the first two decades have following features: (1) both input and output (I/O) measurements are required, and applied basically in the lab environment, (2) FRF or IRF should be estimated as first stage, and modal parameters are then identified, (3) they are based on deterministic framework. Much work has been done in the last decade to further advance MID technology (1) from traditional EMA using I/O measurements to Operational Modal Analysis via Output-only (O/O) data; (2) to estimate modal parameters not only in two-stage approach, but also using I/O measurements directly (one-stage approach); (3) to develop MID algorithm from deterministic to statistical framework to increase estimation accuracy by reducing the influence of the measurement noise and system distortion and provide not only modal parameters but also their confidence intervals.

3 OPERATIONAL vs. TRADITIONAL MID

Experimental modal analysis (EMA), with SISO/SIMO MIMO MID algorithms in time, frequency and spatial domain, has been widely used in trouble shooting, structural dynamics modification, analytical model updating, optimal dynamic design; passive & active vibration control, as well as vibration-based structural health monitoring in aerospace, mechanical and civil engineering. However, traditional EMA has some limitations: (1) Artificial excitation is normally conducted in order to measure FRFs or IRFs. Unfortunately, FRF or IRF are very difficult, or even impossible, to measure in the field testing for large structures; (2) In many industrial applications, the real operation conditions may differ significantly from those for lab testing; (3) Component, instead of complete system, is tested in the lab environment, and boundary condition should be reasonably simulated.

Operational modal analysis (OMA) under ambient excitation has recently drawn great attention in civil engineering. OMA is also very attractive for aerospace and mechanical engineering due to many advantages, such as: (1) Ambient testing is cheap and fast, no elaborate excitation equipment and boundary condition simulation are needed. Traditional modal testing is reduced to be response measurement; (2) Dynamic characteristics of the whole system, instead of component, can be obtained without boundary condition simulation; (3) The model identified under real loading will be linearized due to broad band random excitation at much more representative working points; (4) All or part of measurement coordinates can be used as references; therefore, the identification algorithm used for OMA must be MIMO-type. The closed-spaced or even repeated modes can easily be handled, and suitable for real world complex structures; (5) Operational MID with output-only measurements can be utilized not only for structural control, but also in-situ vibration-based health monitoring and damage identification of the structures.

The challenges encountered in the OMA is that (1) only output data can be used for parameter identification; and (2) the noise/signal ratio in the measured data is much higher than in the controlled experiment in the lab environment.

3.1 Operational MID: TD Techniques

Many time domain MIMO MID algorithms such as PRCE, Extended ITD, ERA and its extension, making use of IRF measurements to extract modal parameters, have successfully been used for traditional EMA. In the 1992's a Natural Excitation Technique (NExT) was proposed [22]. NExT is based on the principle that Correlation Functions (COR) measured under natural, e.g. ambient or operational excitation, can be expressed as a sum of exponentially-decayed sinusoids. Modal parameters, i.e. natural frequency, damping ratio and mode shape coefficient of each decaying sinusoid are identical to the ones of the corresponding structural mode. According to this principle, all the aforementioned TD MIMO identification techniques can be adopted for operational MID by using COR instead of IRFs. The COR functions can be obtained via either Random Decrement technique, inverse Fourier Transform of PSD or directly estimated from random response subjected to broadband natural excitation.

It is worth noticing that free decay data can be applied for operational MID. However, multi-output measurements with respect to one set of initial condition is equivalent to SIMO, but not MIMO system. In order to handle closely spaced or even repeated modes, multiple sets of initial conditions are required in this case.

As mentioned before that System Realization based ERA utilizes IRF for MID. Similar algorithm was available for MID using random response directly based on Stochastic Realization. In 1990's, a powerful tool named as Subspace State-space System Identification (4SID) method [23] is developed, and adopted by modal community afterwards. Stochastic Subspace technique is its special case [24]. Multi-dimensional, or Vector, ARMA mode based approach can also be applied for operational MID in output-only cases. As a kind non-linear identification, Prediction-Error Method is often adopted and initial "guess" for the parameters is required. It makes ARMAV approach computational intensive and rather difficult to use.

All TD MID algorithms have a serious problem in model order determination. Noise or spurious modes are always generated when extracting structural or physical modes. These computational modes are even necessary to account for unwanted effects, such as noise, leakage, residuals and non-linearity's, etc. The computational modes fulfill an important role in that they permit more accurate modal estimation by supplying statistical DOF to absorb these effects. In the traditional MID, IRF can be obtained via inverse FFT of FRF, and may need less computational modes. For operational MID, which makes use of correlation function calculated from random response data, the problems with model order determination and structural modes distinguishing become much more significant. For more effective differentiation between real and spurious modes, many modal validation techniques have been developed. An array of modal indicators, e.g. Modal Assurance Criterion (MAC), Modal Confidence Factor (MCF) and Modal Amplitude Coherence (MAmC), and newly developed Modal Participation Indicator (MPI), etc., were also developed for the purpose [25]. Graphical approach making use of Stability Chart is an effective measure. However, there is, up to now, no guarantee to distinguish structural modes from extraneous ones when deal with complex structure with noisy measurements.

3.2 Operational MID: FD Techniques

On the other hand, classical frequency domain (FD) technique, such as PSD peak picking, has no bother with computational modes and is much faster and simpler to use. However, PSD peak picking technique is inaccurate, especially in mode shape and damping estimation. Moreover, it is very difficult, if not impossible in dealing complex structures with closely spaced modes. A new FD operational MID technique, called Frequency Domain Decomposition (FDD) has been developed to resolve the difficulties encountered when classical FD technique is adopted, while keeps its advantages [25]. FDD is actually a Frequency-Spatial Domain technique, which identifies mode shapes in spatial domain via PSD data as first step. Modal frequencies and damping ratios are then estimated. Theoretically, FDD is an approximate technique. However, based on many successful applications, the reliability and accuracy obtained from FDD is usually better then sophisticated stochastic subspace technique.

4 TWO-STAGE vs. ONE-STAGE MID

The mainstream of the MID is implemented in **two stages**. In FD, FRFs are normally estimated as the first stage, followed by modal parameter estimation as the second stage. For TD algorithm, instead of FRFS, IRFs, Free Decay Response (FDR) or Correlation Functions (COR) are estimated as the first stage. FFT-based FRF estimation makes two-stage MID ease of use. Satisfied accuracy can be reached for there are many ways developed to eliminate bias error and reduce random error in FRF estimation. Two-stage MID is also utilized for output-only OMA, where PSD or COR are estimated in FD or TD, respectively, as the first stage.

FDR can be measured either from transient excitation or sudden termination of board band random excitation; IRF is the counterpart of FRF in TD, and can be calculated via inverse FFT; COR can either be estimated directly from stochastic response, or calculated from PSD via inverse FFT in the output only case, or via Random Decrement (RDD) signature. The latter was explained as free decay response in the beginning, and then proved to be correlation function of the response, and can be computed through many ways from random response of the system. FDR, IRF, COR and RDD can all be expressed as summation of exponentially decayed sinusoids. Each one of these decaying sinusoids has a damped natural frequency and decay rate, which are identical to the one of the corresponding structural mode. Therefore, Time Response Function (TRF) can be defined to represent all these TD features. It should be noted that in frequency domain FRF can only be estimated from both input and output (I/O) data. However, FRT can be obtained with either I/O measurements or output data only (O/O).

A unified two-stage TD MID framework has been developed during a re-visit to modal identification developed with I/O and O/O measurements [26]. The unified approach is based on the formula of modal and system matrices decomposition of TRF, and can cover most of aforementioned TD MID algorithms in both I/O and O/O cases. Numerical accuracy or/and efficiency can be improved via comparisons of different procedures. Implementation issues and numerical considerations, as well as major issues for the two-stage TD MID are also discussed.

MID can also be accomplished in **one-stage** via directly use of I/O or O/O measurements. In fact, two types of main mathematical model for dynamic system, i.e. time series model and state-space model are based on I/O or O/O data.

It was of interest to notice that the paper, published early in 1985 [17] dealing with a unifying approach for DT MID, covered two possibilities via use of IRF data as two-stage approach or direct applying I/O data as one-stage approach. State-space model based MID methods are basically two-stage approach applying Time Response Function (TRF) data. However, Obverser/Kalman filter Identification (ERA/OKID) method [9], expanded from ERA, is a typical one-stage method making use of I/O data directly.

As mentioned before, 4SID method, as a significant advance in system identification, is in essence one-stage approach. In the O/O case, it is called “data-driven” technique, in contrast with correlation or covariance-drive techniques, e.g. NExT and System realization based techniques.

5 STATISTICAL MID vs. DETERMINISTIC MID

In essence most of the deterministic MID methods are actually based on Least Squares Estimation (LSE). The advantages of LSE are simple to implement and fast in computation. However, there is a serious drawback in LSE, i.e. it causes bias error in the estimates. In the LSE, a prediction error, or residual, is assumed when IRF, FDR and CCF are directly used as TRF to form the data matrix. It is well know that LSE would be unbiased only if the prediction error is white noise. In reality, the error would never be such a white noise; even the system corrupted only by white output or measurement noise! Therefore, bias error caused by color noise of the prediction error becomes one major problem in the LSE-based MID. There are two possible ways to overcome bias problem caused by LSE: one is to properly model the noise (noise modeling methods); the other is to eliminate bias error without noise modeling.

Actually noise modeling is not only adopted to deal with measurement noise but also to compensate leakage, residuals and non-linearity. Many issues still remain to be explored. Noise modeling could bring lots of new problems, e.g. noise model selection, model order determination, and iteration convergence, etc. There are other methods available to reduce or eliminate bias error introduced by LSE, for example, the methods via Instrument Variable (IV), Double LS (DLS), Total LS (TLS) and LS with data correlation.

Most MID methods are developed within deterministic framework, which creates many serious issues. For example, little attention has been paid for identification error, random and bias, analysis. The accuracy of the identified modal parameters is often questionable, especially with noisy measurements. For the system identification, or parameter estimation, statistical property is often of great importance. These statistical properties are: (1) Consistency, the estimator should converge, in probability, to the true value, i.e. small bias error; (2) Efficiency, the estimator should have small uncertainty (i.e. random error) on the estimates; (3) Robustness, the estimator keeps consistent and efficient when the assumptions made in its construction are no longer valid, among others.

MID based on statistical framework has advanced in both frequency domain (FD) and time domain (TD). A FD MIMO statistical framework was developed in 1990's based on transfer function model with Matrix-Fraction Description (MFD). Different implementations were proposed based on different Cost Functions [27]. Maximum Likelihood Estimator (MLE) algorithm is a typical statistical approach using likelihood function as cost function. The deterministic Least Squares Estimator (LSE) can be adopted into statistical framework by introducing a covariance matrix into the cost function, and implemented as Weighted Generalized Total LSE (also called Bootstrapped Total LSE), Iterative Weighted Total LSE, Weighted Nonlinear LSE, as well as Logarithmic Weighted LSE.

MLE can take noise information into account; and, therefore, it performs better than deterministic ones, especially when the measurements are very noisy. An additional advantage of the statistical approach is that it is possible to derive confidence Intervals for the estimated modal parameters almost without any additional calculation. FD MLE has been extended to utilize orthogonal polynomials [28].

Disadvantages of statistical MID approaches are (1) much more computational intensive, (2) not suitable to handle large amount of data, and (3) requiring initial guesses for the parameters to be estimated, which need to be identified in advance via deterministic counterpart. In the late of 1990's, the MLE algorithm has been modified and optimized to resolve these issues. It is noticed that aforementioned FD statistical MID approaches can be implemented in two or one-stage with either I/O or O/O data.

There are two major directions for TD MID under statistical framework: Subspace State-space System Identification (4SID) methods and ARMRX methods. Innovation State-space model is utilized when both system error and measurement error are taking account. Good accuracy for parameter estimation can be obtained by stochastic 4SID method. Compared to its deterministic counterpart, stochastic 4SID is computational intensive. Statistical TD MID approaches are basically one-stage methods. PEM estimator is a consistent and statistically efficient for a certain choice of cost function. However, it is very computational intensive and very time consuming. PEM-ARMAX method is hardly to deal with large dimension system. A Linear Multi-Stage (LMS) ARMAX method for effective MIMO MID in the presence of noise is proposed [29]. The LMS-ARMAX method overcomes many of the difficulties that had rendered MIMO ARMAX identification in use for complex structures.

6 CONCLUDING REMARKS

Major developments of modal identification (MID) in the first two decades were briefly summarized. The latest developments and issues of MID are then presented, which cover (1) Traditional Experimental Modal Analysis (EMA) using both Input and Output measurements and Operational Modal Analysis using Output data only; (2) Two-stage modal identification and one-stage modal identification, and (3) Deterministic modal identification and statistical modal identification, which takes measurement noise and system uncertainty into account.

Modal identification has obtained substantial progress in the past three decades. A variety issues have been dealt with. However, it seems not matured yet. Much work still remains to be done. A unified framework in wider sense is need to be further developed, which should cover traditional experimental modal analysis with input & output (I/O) measurements and operational modal analysis with output data only (O/O); statistical and deterministic point of views in both time and frequency domain.

The publications on MID in the last three decades are overwhelming. The papers sighted in this overview are within author's knowledge. They are far from complete and might neither be comprehensive nor most representative. The author would like to express his apology in advance for missing important contributions.

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