APPLICATION OF THE
RANDOM DECREMENT TECHNIQUE
IN OPERATIONAL MODAL ANALYSIS

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Abstract

The random decrement (RD) technique is a time domain procedure, where the structural responses to operational loads are transformed into random decrement functions, which are proportional to the correlation functions of the system operational responses or can, equivalently, be considered as free vibration responses. This paper presents a general review on the application of the RD technique in operational modal analysis. It refers its application in association with time domain modal identification methods, like the Ibrahim time domain method or the stochastic subspace covariance driven method. It also describes the use of the RD technique in association with frequency domain output-only modal identification methods, like the frequency domain decomposition method (FDD). A practical example is given showing the results obtained from ambient vibration tests performed in a civil engineering structure.

1   Introduction

The random decrement (RD) technique was proposed by Cole in the end of the sixties and beginning of the seventies (ref [2, 3]) within his work at NASA concerning the analysis of the dynamic response of space structures subjected to ambient loads. The main purpose of that work was the identification of the dynamic characteristics and in-service damage detection of space structures from the measured response. Cole was looking for a simple and direct method for translating the response time history into a form more meaningful to the observer (ref [2]). The development of the RD technique was motivated by the problems encountered in damping estimation from the half power bandwidth of the spectral densities estimated using the FFT algorithm, and by the lack of an approach to detect non-linearities from the spectral densities of the response to ambient loads.

Since its initial developments (ref [2, 3]), the RD technique has been extensively investigated (ref [5, 6, 8, 9, 10, 11, 12, 13, 15, 17]). It consists in a rather simple process of averaging time segments of the measured structural responses, with a common initial or triggering condition. By using that procedure, the responses are transformed into RD functions.

At first (ref [2]) the RD functions were interpreted as free vibration responses of a system, which is the natural interpretation considering the argument that was initially presented for the validity of the technique, but latter (ref [10, 11, 13]) it has been proved that, under the assumption that the analysed responses are a realization of a zero mean stationary Gaussian stochastic process, the RD functions are proportional to the correlation functions of the responses and/or to their first derivatives in relation to time.

Most of the applications of the RD technique have been developed in association with time domain modal identification methods, like the Ibrahim time domain method (ITD) (ref [4]) or the eigensystem realization algorithm (ERA) (ref [7]) which is equivalent to the covariance driven stochastic subspace method (SSI-COV) (ref [14]). This is probably a natural consequence of the fact that the RD technique is a time domain procedure and therefore it is natural to consider it in association with the time domain modal identification methods.

The RD technique can also be used for the purpose of spectral estimation (ref [9]) and frequency response function estimation (ref [12]) and therefore it can be applied in association with the frequency domain modal identification methods. For that purpose, the spectral densities of a system response are estimated from the FFT of the RD functions and not by the more usual procedure of averaging the FFT of windowed samples, eventually overlapped, of the response time histories, known as ensemble averaging or Welch method (ref [1]). This approach of using the spectral densities estimated from the FFT of the RD functions, in the frequency domain output-only modal identification methods, has been applied for a simple example (ref [17]) and showed considerable advantages in relation to using the spectral densities estimated with the more usual procedure.

A scheme showing several different possible ways to apply the RD technique in operational modal analysis is presented in Figure 1.
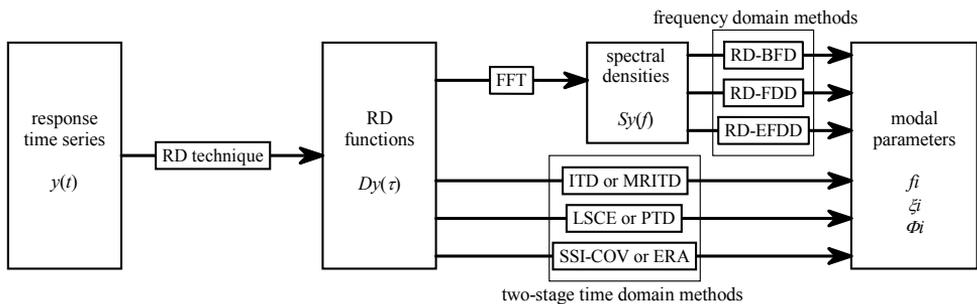

Figure 1 Application of the RD technique in operational modal analysis

The following sections of this paper will present a more detailed description of the RD technique and of its application with time domain and frequency domain modal identification methods. In the end, an example will be used to illustrate the application of the different methods.

2   General Description of the RD Technique

The concept and argument for the validity of the RD technique presented by Cole (ref [2]) is not difficult to understand if one thinks that the response of a system to random input loads is, in each time instant t, composed by three parts: the response to an initial displacement; the response to an initial velocity; and the response to the random input loads between the initial state ant the time instant t. By averaging a large number of time segments of the response with the same initial condition, the random part of the response will have a tendency to disappear from the average, and what remains is the response of the system to the initial conditions. From this simple explanation the interpretation of the RD functions as free vibration responses is immediate.

Another effect of the time segments averaging of the RD technique is the noise reduction in the resulting RD functions. This is an important advantage since the experimentally measured structural responses always have some noise content.

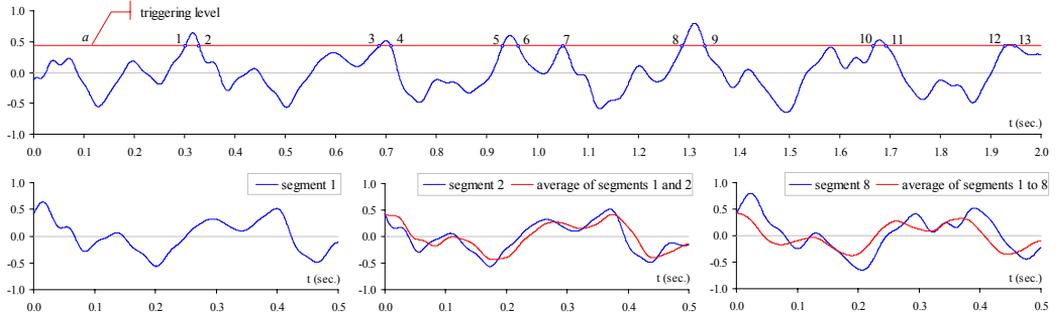The basic concept of the RD technique is illustrated in Figure 2.

Figure 2 Basic concept of the RD technique

The initial idea of the RD technique involved only auto RD functions, where the triggering condition and the time segments to be averaged are defined in the same response signal. Latter, the concept of cross RD functions was proposed (ref [4]) where the triggering condition is defined in one response signal and the time segments to be averaged are taken from the other simultaneous response signals. One can therefore evaluate a complete matrix of RD functions, or a reference-based approach can be adopted, where only a few reference response measurements are considered to apply the triggering conditions.

Considering two response time histories $x(t)$ and $y(t)$, simultaneously measured, the auto $D_{XX}(\tau)$ and cross $D_{XY}(\tau)$ RD functions can be mathematically defined by the following expressions:

$$D_{XX}(\tau) = \frac{1}{N} \sum_{i=1}^{N} x(t_i + \tau) | Tx(t_i) \qquad\qquad D_{XY}(\tau) = \frac{1}{N} \sum_{i=1}^{N} y(t_i + \tau) | Tx(t_i) \qquad (1)$$

where N is the number of averaged time segments and $Tx(t_i)$ is the triggering condition, in this case, applied to time history $x(t)$.

For the evaluation of the RD functions it is possible to consider different triggering conditions $Tx(t_i)$ (ref [13, 15]), which can also be interpreted as special cases of a generalized triggering condition (ref [13]). The most common triggering conditions considered in the RD technique are:

   a) level crossing (the one illustrated in Figure 2);

$$Tx(t_i) = \{x(t_i) = a\} \qquad (2)$$

   b) positive points;

$$Tx(t_i) = \{a \le x(t_i) < b\} \qquad (3)$$

   c) zero crossing with positive slope;

$$Tx(t_i) = \{x(t_i) = 0, \dot{x}(t_i) > 0\} \qquad (4)$$

   d) local extremum.

$$Tx(t_i) = \{a \le x(t_i) < b, \dot{x}(t_i) = 0\} \qquad (5)$$

In the scheme presented in Figure 2, the triggering points are defined in the positive part of the response time history. It is also possible to apply the triggering condition in the negative part of the response signal and to average, with a change in the sign, the time segments thus extracted with the ones extracted from the positive part of the response time history. The benefit of using this procedure is that the number of triggering points increases considerably (roughly, it doubles), which is advantageous for the desired effects of the time segments averaging of the RD technique.

One important aspect of the application of the RD technique is the definition of the triggering levels, a and b in equations (2) and (3). It is important to note that, in general, it is advantageous to use a large number of triggering points, however the time series points with low values might be more contaminated by noise than the points with large values. Therefore, an equilibrium must be achieved between having a large number of triggering points and having a high triggering level. A usual and good option is to adopt the positive points triggering condition considering the levels $a = \sigma_x$ and $b = \infty$, where σ_x is the standard deviation of the response time history under analysis. It is also possible to define optimum triggering levels, in the sense of minimization of the variance of the estimate of the RD functions (ref [13]). In the case of the level crossing triggering condition, this optimum level is $a = \sqrt{2}\,\sigma_x$.

Another important point of the RD technique is the length of the time segments that are extracted from the response time histories. For application of the time domain modal identification methods the RD functions must have enough points to allow an over-determination of the equations from which the system matrices are estimated (considering their maximum order). For application of the frequency domain modal identification methods it is important that the RD functions are evaluated with sufficient time length to have a complete decay within that length (in general, this corresponds to a larger time length than the one required for the time domain methods).

It has already been mentioned that, under the assumption that the analysed responses are a realization of a zero mean stationary Gaussian stochastic process, the RD functions are proportional to the correlation functions of the responses and/or, depending on the triggering condition, to their first derivatives in relation to time (ref [10, 11, 13]). The RD technique can therefore be considered as a method to estimate the correlation functions (ref [10]). For this purpose, the RD technique is an efficient computational procedure. Comparisons have been made (ref [10]) of different techniques that can be used to estimate the correlation functions, including the direct method, the FFT based method and the RD technique, showing that the RD technique is faster than the direct method and in many situations faster than the FFT based one (eventually for long estimates of the correlation functions the FFT based method is more competitive).

The auto RD functions are in general known to have less noise than the cross RD functions (ref [15]). To circumvent this problem, one technique that has been proposed is the vector triggering random decrement (VRD) technique, where the triggering condition involves simultaneously two or more of the response time histories (ref [13]).

3   Application of the RD Technique with Time Domain Methods

Traditionally, the RD technique has been used in association with time domain modal identification methods like the Ibrahim time domain method (ITD) (ref [4]) or the eigensystem realization algorithm (ERA) (ref [7]) which is equivalent to the covariance driven stochastic subspace method (SSI-COV) (ref [14]). Other methods can also be used like, as indicated in Figure 1, the multiple reference version of the Ibrahim time domain method (MRITD), the least squares complex exponential method (LSCE) and the poly-reference time domain method (PTD). All these methods can be considered in a unified approach, as two-stage time domain modal identification methods

(ref [16]). They can be applied, in an equivalent way, to correlation functions, impulse response functions, free vibration records or RD functions, since all these time response functions (ref [16]) can be factorized in a similar manner. Considering the matrices of the state-space representation of a discrete-time dynamic system, the formulation of these time domain modal identification methods can be derived from the following factorization of the correlation functions R (ref [14]):

$$R_k = C \, A^{k-1} \, G \tag{6}$$

where C is the discrete output matrix, A is the discrete state matrix and G is the next-state output correlation matrix.

The time domain methods MRITD and SSI-COV will be briefly presented bellow. For this purpose it is convenient to consider the correlation functions arranged in the following two Hankel matrices (note that matrix H_1 is shifted in time by one time interval in relation to matrix H_0):

$$H_0 = \begin{bmatrix} R_1 & R_2 & R_3 & \cdots & R_q \\ R_2 & R_3 & R_4 & \cdots & R_{q+1} \\ R_3 & R_4 & R_5 & \cdots & R_{q+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_p & R_{p+1} & R_{p+2} & \cdots & R_{p+q-1} \end{bmatrix} \quad H_1 = \begin{bmatrix} R_2 & R_3 & R_4 & \cdots & R_{q+1} \\ R_3 & R_4 & R_5 & \cdots & R_{q+2} \\ R_4 & R_5 & R_6 & \cdots & R_{q+3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & R_{p+2} & R_{p+3} & \cdots & R_{p+q} \end{bmatrix} \tag{7}$$

3.1 Multiple Reference Ibrahim Time Domain Method (MRITD)

The MRITD method consists basically in a least squares adjustment using the correlation functions estimated from the response time histories. In this method the discrete state matrix A is estimated from the following equation:

$$A \, H_0 = H_1 \quad \Leftrightarrow \quad A = H_1 \, H_0^+ \tag{8}$$

where H_0^+ is the pseudo-inverse of matrix H_0, which can be computed using several different procedures like singular value decomposition (SVD) or orthogonal decomposition (QR).

3.2 Covariance Driven Stochastic Subspace Identification (SSI-COV)

The SSI-COV method is based on the factorization of the Hankel matrix H_0 in the product of the observability matrix O_p by the stochastic controllability matrix Γ_q (ref [14]):

$$H_0 = O_p \, \Gamma_q \tag{9}$$

In practice, the factorization expressed in equation (9) is achieved by applying the SVD algorithm to matrix H_0 and the matrices O_p and Γ_q can be obtained from the resulting factors:

$$H_0 \xrightarrow{SVD} U_{Hs} \, \Sigma_{Hs} \, V_{Hs}^T \quad \Rightarrow \quad \begin{cases} O_p = U_{Hs} \, \Sigma_{Hs}^{1/2} \\ \Gamma_q = \Sigma_{Hs}^{1/2} \, V_{Hs}^T \end{cases} \tag{10}$$

Matrices C and G, see equation (6), can be extracted from O_p and Γ_q determined from equation (10). The discrete state matrix A can be determined using several procedures, one of which is:

$$A = \Sigma_{Hs}^{-1/2} \, U_{Hs}^T \, H_1 \, V_{Hs} \, \Sigma_{Hs}^{-1/2} \tag{11}$$

4 Application of the RD Technique with Frequency Domain Methods

There are basically three frequency domain output-only modal identification methods: the basic frequency domain method (BFD) or peak picking method (PP); the frequency domain decomposition method (FDD); and the enhanced frequency domain decomposition method (EFDD). The common data for these methods are the estimates of the spectral density functions of the response of a structural system. Usually, those estimates are obtained using a procedure that can be attributed to Welch (ref [1]) and consists in: division of the response records in several, eventually overlapped, segments, whose size determines the frequency resolution of the spectral estimates; application of a signal processing window in order to reduce the effects of leakage; computation of the DFT of the windowed data segments trough the use of the FFT algorithm; computation of averaged auto and cross spectra considering the DFT's of the data segments.

Once the estimates of the spectral density functions are evaluated, the procedures to analyse them, in order to extract the modal properties of a system, are slightly different in each of the methods BFD, FDD and EFDD.

In the BFD method the auto-spectra are normalized and averaged in order to obtain an averaged normalized power spectral density function (ANPSD) that, in principle, shows all the resonance peaks corresponding to the vibration modes of a system. Identification of the frequencies of those peaks gives a first idea about the frequencies of the vibration modes of a system. Further analysis is needed of the coherence function and also of the amplitude and phase relations between the records obtained along the different experimental degrees of freedom. Both the coherence function and the amplitude and phase relations are evaluated with the elements of the spectral density functions matrix. At the frequencies of the natural vibration modes of a system, the coherence function should present values close to 1. The amplitude and phase relations between the different degrees of freedom are evaluated with the H_1 estimate of the transmissibility frequency response function and can be considered as an estimate of the modal components, from which the mode shapes of a system can be constructed.

In the FDD method the spectral density functions matrix is, at each discrete frequency, decomposed in singular values and vectors using the SVD algorithm. By doing so, the spectral densities are decomposed in the contributions of the different modes of a system that, at each frequency, contribute to its response. In each frequency, the dominant mode shows up at the 1st singular value spectrum and the other modes at the other singular values spectra. From the analysis of the singular values spectra it is therefore possible to identify the auto power spectral density functions corresponding to each mode of a system, which may include parts of several singular values spectra, depending on which mode is dominant at each frequency. In the FDD method, the mode shapes are estimated as the singular vectors at the peak of each auto power spectral density function corresponding to each mode.

The EFDD method is closely related with the FDD technique, with only some additional procedures to evaluate the damping and to get enhanced estimates of the frequencies and mode shapes of a system. In the EFDD method, the analysis of the singular values spectra, takes a further step forward. The selection of the auto-spectra corresponding to each mode of a system is performed based on the values of the MAC coefficient between the singular vectors at the resonance peaks and at their neighbouring frequency lines. Those SDOF auto-spectral density functions are then transformed back into the time domain by inverse FFT, resulting in auto-correlation functions for each mode of a system. Enhanced estimates of the frequencies of the modes of a system are obtained from the zero crossing times of those auto-correlation functions. The damping coefficients are estimated from the logarithmic decrement of those auto-correlation

functions. Finally, the estimate of the mode shapes is also enhanced, considering all the singular vectors within each SDOF auto-spectral density function, weighted with the corresponding singular values.

It has been referred before that the RD functions can be interpreted as free vibrations of a system, therefore a matrix of RD functions can be looked as a set of records from free vibration tests of a system; each column or line of the matrix corresponding to a different test where initial conditions are imposed at the corresponding degree of freedom and the response is measured in all the degrees of freedom. Thus, it seems reasonable to evaluate the spectra of the RD functions, using the FFT algorithm, and to apply the frequency domain output-only modal identification methods to the spectral density functions obtained in such way. It is however necessary to take into account the problems associated with the discrete Fourier transform, namely the effects of leakage. To avoid those effects, the RD functions must be computed with a total length that allows them to have a complete decay within that length. If this condition is fulfilled then the FFT algorithm can be applied directly to the RD functions, without the need to use signal-processing windows.

Notice that the requirement of having RD functions with a complete decay within their length is not an important condition for the time domain methods, but it is an indispensable one for the estimation of the spectral densities as the Fourier transform of the RD functions.

An averaged spectral density functions matrix can be evaluated as the mean of the spectral matrices computed from each column or line of the RD functions matrix. The three frequency domain output-only modal identification methods can then be applied to that averaged spectral density functions matrix, in a similar manner as they are applied to the spectral densities estimated by the more usual procedure. The resulting identification methods can be named as RD-BFD, RD-FDD and RD-EFDD since they correspond to a combination of the RD technique with the methods BFD, FDD and EFDD. There are advantages in doing this combination which are a consequence of the noise reduction from the averaging of time segments that is performed in the RD technique, and also from avoiding the effects of leakage (if the RD functions are evaluated with enough length to have a complete decay within that length).

5 Practical application in a railway bridge

In this section, the RD technique and two modal identification methods, that can be used in association with it, will be applied to the data obtained in ambient vibration tests performed in a railway bridge, Corona bridge (see Figure 3), located in the South of Portugal. This bridge has a pre-stressed concrete box girder deck with a total length of 208 m, divided in three intermediate spans with 48 m and two extreme spans with 32 m (see Figure 4).

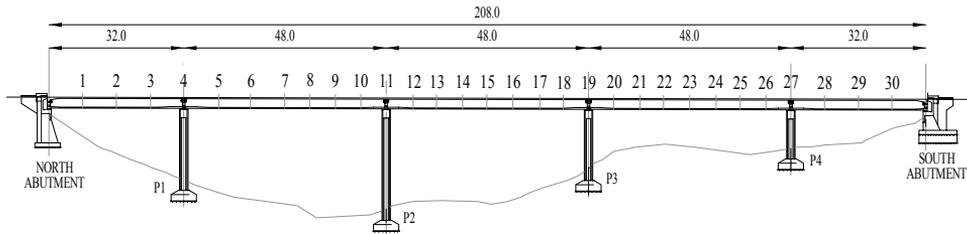Figure 3 General views of Corona bridge

Figure 4 Elevation view of Corona bridge

The ambient vibration tests performed in Corona bridge were conducted by LNEC in September 2004, as part of tests performed after the end of its construction. At the time they were made, there was already railway traffic crossing the bridge and therefore some records of the acceleration response of the structure under the effects of railway traffic were also obtained. However, for the results that are presented in this paper, only the ambient vibration data is analyzed, which corresponds to situations without any train crossing the bridge.

The ambient vibration tests of Corona bridge were performed with 15 Kinemetrics ES-U force balance accelerometers, signal conditioning equipment constructed at LNEC and data acquisition hardware and software from National Instruments. For those tests, the equipment was configured for a sensitivity of 2 Volt/mg, and the ambient vibration data was acquired during about 22 minutes using a sampling frequency of 1000 Hz. The records obtained in this way were later pre-processed, with high-pass digital filtering at 0.2 Hz with a 1 pole Butterworth filter, low-pass digital filtering at 10 Hz with an 8 poles Butterworth filter and decimation to a sampling frequency of 25 Hz.

The tests were conducted in three set-ups and vertical and transverse accelerations were measured in 30 sections of the bridge deck, whose location is indicated in Figure 4. Only the analysis of the vertical records is presented in this paper.

Using the RD technique and considering a level crossing triggering condition, with the optimal value of the triggering level as referred previously, the RD functions were evaluated for the accelerations measured in the three set-ups of the tests. Segments with a time length of 1024 values at 25 Hz (about 40 seconds) were considered for that evaluation. An auto and a cross RD function corresponding to the vertical records obtained in the third set-up in sections 23 and 15, are represented in Figure 5.
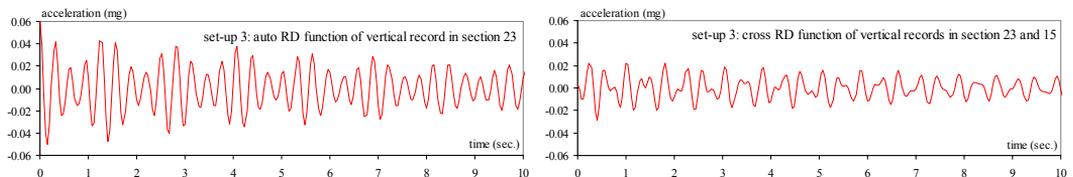

Figure 5 Examples of auto and cross RD functions

The estimated RD functions were considered to apply the SSI-COV method and the RD-FDD method. Figure 6 shows the stabilization diagram corresponding to the application of the SSI-COV method to the RD functions estimated with the data of the third set-up and the averaged singular values spectra corresponding to the application of the RD-FDD method. The frequencies and mode shapes identified with the two methods SSI-COV and RD-FDD are presented in Figure 7.
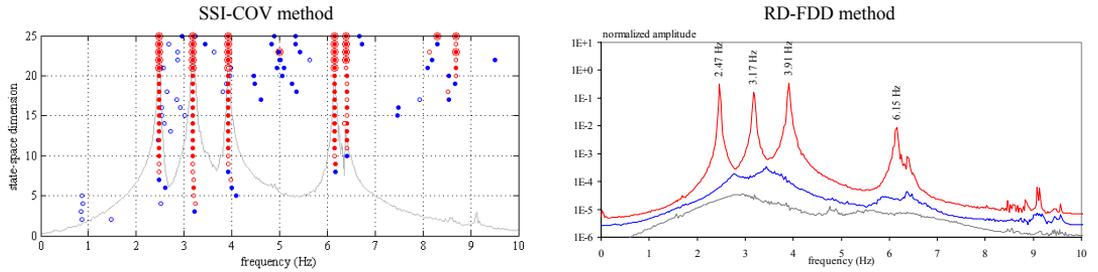
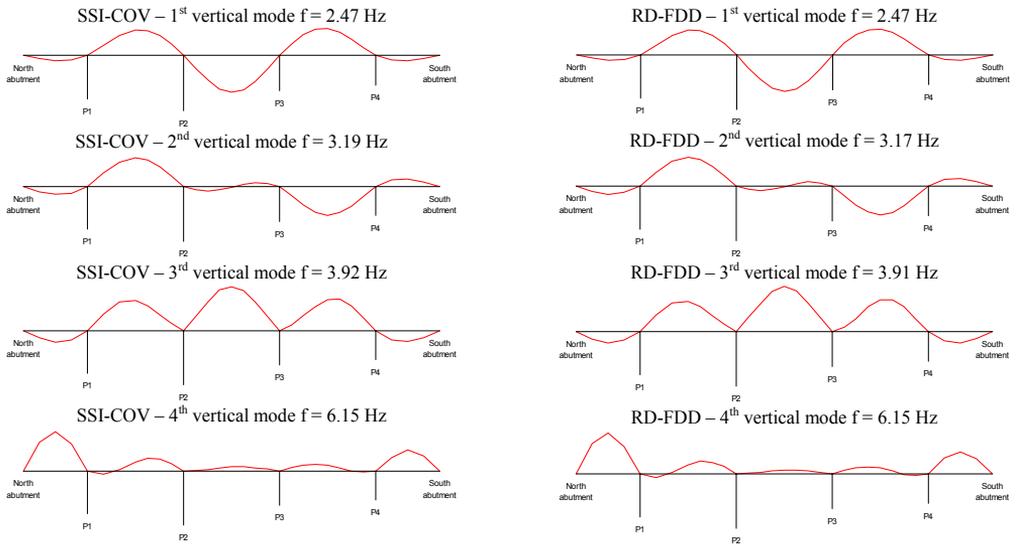Figure 6 Stabilization diagram and averaged singular values spectra

Figure 7 Identified frequencies and mode shapes

6 Conclusions

A general review on the application of the random decrement technique in operational modal analysis was made in this paper. Traditionally, the technique has been used in association with time domain modal identification methods. However, it can also be used in association with frequency domain output-only modal identification methods, with advantages in relation to the usual procedure that is used in these methods to estimate the spectral density functions.

Considering the data obtained in ambient vibration tests conducted in a railway bridge, a practical example was presented, illustrating the application of the random decrement technique with a time domain method, the stochastic subspace covariance driven identification method, and with a frequency domain method, the frequency domain decomposition method.

7 References

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