Determination of Stress Histories in Structures by Natural Input Modal Analysis

Henrik P. Hjelm
Structural Vibration Solutions A/S
Novi Science Park, Niels Jernes Vej 10, DK-9220 Aalborg East, Denmark
E-mail: h.hjelm@stofanet.dk

Rune Brincker
Department of Building Technology and Structural Engineering
Aalborg University, Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark
E-mail: i6rb@civil.aau.dk

Jesper Graugaard-Jensen
Structural Vibration Solutions A/S
Novi Science Park, Niels Jernes Vej 10, DK-9220 Aalborg East, Denmark
E-mail: jespergj@stofanet.dk

Kasper Munch
Structural Vibration Solutions A/S
Novi Science Park, Niels Jernes Vej 10, DK-9220 Aalborg East, Denmark
E-mail: k_munch@stofanet.dk

Abstract

In this paper it is shown that stress histories can be estimated with high accuracy by integrating measured accelerations to obtain displacement and then performing a modal decomposition of the so estimated displacements. The relation between the modal coordinate and the stress in an arbitrary point is established by a finite element model. It is shown, that when performing the modal decomposition, either experimental mode shapes or numerically estimated mode shapes can be used. However, it is important to verify that the numerical modes correspond reasonably well to the experimental modes. It is shown that the so estimated stress histories can replace strain gauge measurements in many cases, and it allows for an accurate estimation of fatigue damage.

Nomenclature

$\mathbf{\Phi}$	Mode shape matrix.
$\Delta\sigma$	Stress range.
D	Damage.
f	Natural frequency.
N	Number of stress cycles.
$\mathbf{q}(t)$	Time depend modal coordinate vector.
$\mathbf{y}(t)$	Time depend displacement vector.
$_{exp}$	Experimental determined value.
$_{FE}$	Numerical (finite element) determined value.

1. Introduction

Determination of stress histories in dynamic sensitive structures is usual assigned with large uncertainty, since a conventional determination of stress histories requires information about e.g. the load history and transfer function – both assigned with some uncertainty. Minimizing these uncertainties can simply be done by determine the stresses experimentally. Experimentally determined stresses are normally determined from strains measured with strain gauges, but fatigue sensitive joints are often located in sections where mounting of strain gauges are difficult or even impossible, e.g. below water on offshore structures. Furthermore strain gauges are not reliable for long time measurements and are, for this reason, inapplicable for determination of the stress histories.

By determining the stress histories in structures by natural input modal analysis two important advantages are introduced. First, the method is based on measurements with accelerometers, which are known as reliable for long time measurements. Second, by introducing a finite element model, the stress history can be calculated in any arbitrary point of a structure when accelerations are measured in only a few points of the structure.

The theory of determination of stress histories by natural input analysis is explained in details and validated through experiments in Graugaard-Jensen et al. [1]. In this paper the theory is summarized and the main results of the experiments are shown. More results can be found in Graugaard-Jensen et al. [2].

2. Theory

The accelerations of a structure, exposed to a stochastic loading, are measured in a few easily accessible points of the structure. A modal identification is performed to obtain the natural frequencies f_{exp} and mode shapes $\mathbf{\Phi}_{\text{exp}}$ of the structure. The identification may be performed by e.g. Stochastic Subspace Identification (SSI), cf. Van Overschee and De Moor [3], or Frequency Domain Decomposition (FDD), cf. Brincker et al. [4]. The FDD is used in this paper as implemented in the ARTeMIS Extractor software. The FDD is based on calculation of Spectral Density Matrices of the measured data series by discrete Fourier transformation. For each frequency line the Spectral Density Matrix is decomposed into auto spectral functions corresponding to a single degree of freedom system (SDOF).

A finite element model is calibrated to obtain

$$\mathbf{\Phi}_{\text{exp}} = \mathbf{A}\,\mathbf{\Phi}_{\text{FE}} \qquad (1)$$

where $\mathbf{A}$ is an observation matrix containing zeros and ones.

The measured accelerations are integrated twice to obtain the displacements $\mathbf{y}_{\text{exp}}(t)$. The integration is performed by use of Simpson's Rule, cf. Kreyszig [5], and the resulting numerical drift is removed by digital high-pass filtering, e.g. by use of Butterworth filters.

The obtained displacements $\mathbf{y}_{\text{exp}}(t)$ are expanded in modal coordinates $\mathbf{q}(t)$ by use of either the experimental mode shapes $\mathbf{\Phi}_{\text{exp}}$

$$\mathbf{y}_{\text{exp}}(t) = \mathbf{\Phi}_{\text{exp}}\,\mathbf{q}(t) \qquad (2)$$

or numerical mode shapes $\mathbf{\Phi}_{\text{FE}}$

$$\mathbf{y}_{\text{exp}}(t) = \mathbf{A}\,\mathbf{\Phi}_{\text{FE}}\,\mathbf{q}(t) \qquad (3)$$

From the modal coordinates and the numerical mode shapes, the response $\mathbf{y}_{\text{FE}}(t)$ in any arbitrary point of the structure is simply calculated by

$$\mathbf{y}_{\text{FE}}(t) = \mathbf{\Phi}_{\text{FE}}\,\mathbf{q}(t) \qquad (4)$$

and the strains and far field stresses can be calculated in any point of the structure by traditional finite element calculations. The hot spot stresses are calculated by applying the finite element relationship between the far field stresses and the hot spot stresses.

Use of the experimental mode shapes in the modal expansion may seem as the most obvious, since the response is experimental determined, but in some cases equation (1) cannot be fulfilled. If e.g. the structure is symmetric, some of the identified mode shapes may be unstable and the direction may differ from the numerical mode shapes. In such a case, the modal expansion must be performed by use of the numerical mode shapes. However, it is important that the FE-model is always correct calibrated. The calibration is checked by comparison the experimental and numerical natural frequencies and by calculating the Modal Assurance Criteria (MAC) between the experimental and numerical mode shapes. The MAC varies between zero and one, where one denotes full correlation of the modes. The MAC between the *i*th experimental and numerical mode shape is calculated as

$$\mathrm{MAC}(\Phi_{\exp,i}, \Phi_{\mathrm{FE},i}) = \frac{\left|\Phi_{\exp,i}\Phi_{\mathrm{FE},i}\right|^2}{(\Phi_{\exp,i}^T\Phi_{\exp,i})(\Phi_{\mathrm{FE},i}^T\Phi_{\mathrm{FE},i})} \tag{5}$$

The MAC between $\mathbf{\Phi}_{\exp}$ and $\mathbf{\Phi}_{\mathrm{FE}}$ should generally be high to obtain accurate results, unless $\mathbf{\Phi}_{\exp}$ is unstable.

If the number of mode shapes of the system equals the measured degrees of freedom, then equation (2) or (3) is solved directly. If the number of measured degrees of freedom exceeds the number of mode shapes, the equation is over determined and is solved by linear regression, e.g. the Least Square method.

If the number of mode shapes exceeds the number of measured degrees of freedom the equation is under determined and must be divided into sub-systems. The dividing into sub-systems may be performed by digital filtering or directly in frequency domain by considering the same problem frequency range by frequency range. The division into sub-systems shall furthermore ensure that mode shapes with high correlation is not included in the same sub-system. High correlated mode shapes included in the same sub-system may result in error on the calculated modal coordinates, resulting in large errors on the following calculation of the stress history, cf. Graugaard-Jensen et al. [1]. The correlation is checked by calculating the MAC.

3. Introduction to Experiments

Two experiments are performed; an experiment on a laboratory structure and an experiment on a lattice tower, cf. Figure 1. The laboratory structure is a 2 m high cantilever beam with a mounted beam and weight on the top to induce torsion modes. The purpose of the experiment is to demonstrate that the stress history in the structure can be calculated in any point with sufficient accuracy. This is demonstrated by simultaneous measuring strains in two sections on the lower part of the structure and accelerations on the upper part of the structure. The stresses in the lower part of the structure are subsequently calculated from the strain gauge measurements and from the acceleration measurements and the stress histories are compared.

For simplicity, in the rest of this paper the stress calculations based on the strain gauges measurements are referred to as “measured stresses”, and the stress calculations based on the acceleration measurements (natural input analysis) are referred to as “calculated stresses”.

The lattice tower, located near the Structural Research Laboratory of Aalborg University, is a 20 m high welded structure with a constant width of 0.9 m. Plywood plates are mounted on the upper 1.5 m of the structure to increase the wind load. The purpose of the experiment on the lattice tower is to verify that the method produces satisfying results on a real structure exposed to stochastic loading. Strains and accelerations are measured using same approach as used for the laboratory structure and the measured and calculated stresses are compared. Furthermore, the uncertainty on the calculated stress history is defined by comparison of the measured and calculated damage.

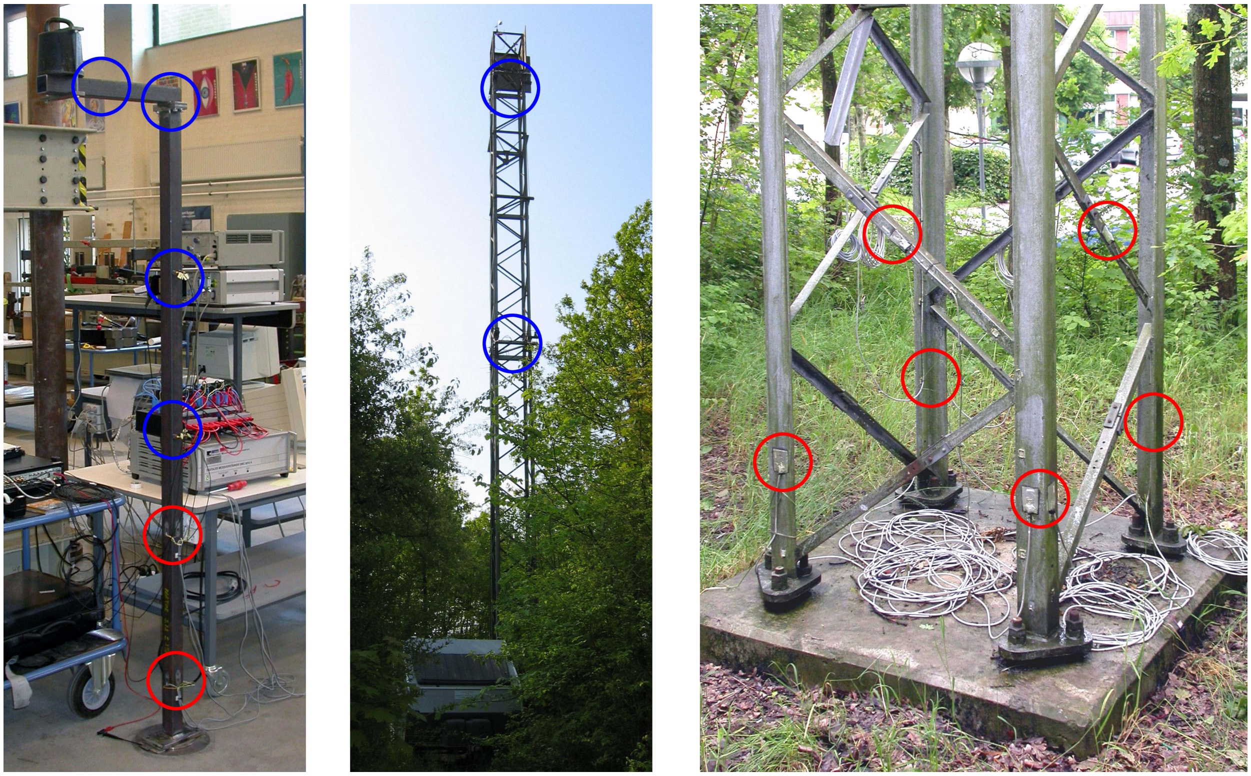

Figure 1 *(Left) Laboratory structure, (center) lattice tower, (right) base of lattice tower. Red and blue circles indicate location of strain gauges and accelerometers respectively.*

4. Laboratory Experiment

4.1. Test Program for Laboratory Experiment

Accelerations are measured in four sections on the upper part of the structure. In each section the measurements are performed in three points, making it possible to determine rigid-body motion and torsion. The strains are measured in two sections on the lower part of the structure with single 120 Ω strain gauges. The strain gauges are mounted on each of the four sides of the profile 0.2 m and 0.7 m above the support, measuring in the axial direction of the profile. The dynamic load on the structure is generated by tapping on the structure with a rubber hammer, and random blows are applied all over the structure to excite all modes.

The pilot experiment is run in two stages. In stage one the modal properties of the structure is determined and only accelerations are sampled. Accelerations are sampled with 512 Hz in 500 sec in two sections (six channels) at a time. In stage two both accelerations (six channels) and strains (eight channels) are sampled with 512 Hz in one operation in approximately 10 sec. These measurements are subsequently used for comparison of measured and calculated stresses.

The structure is modeled in the finite element program ANSYS by 20-node isoparametric elements with three degrees of freedom (d.o.f.) per node. The model is calibrated by applying springs in the joints in the model, so that the numerical mode shapes $\mathbf{\Phi}_{FE}$ correspond reasonably well to the experimental mode shapes $\mathbf{\Phi}_{exp}$.

4.2. Data Processing and Results of Laboratory Experiment

In Figure 2 the results of the modal identification by FDD is plotted. Nine modes are identified. In Table 1 the experimental and numerical determined natural frequencies are listed and compared, and Modal Assurance Criteria (MAC) between $\mathbf{\Phi}_{FE}$ and $\mathbf{\Phi}_{exp}$ are calculated. The high values of MAC indicate that the FE-model is well calibrated. In Figure 7 the mode shapes are plotted.

The modal decomposition is performed using $\mathbf{\Phi}_{exp}$, cf. equation (2), and the response is calculated in the finite elements holding the coordinates to the strain gauges by use of equation (4). The stresses in these elements are subsequently calculated by traditional finite element calculations, cf. Cook et al. [6].

The numerical drift resulting from the integration is removed by an 8^{th} order 1 Hz low-pass Butterworth filter.

Since nine modes are presented, but accelerations are measured in only six channels at a time, the equation system is under determined and must be split into two or more sub-systems. It is chosen to split the equation system into four sub-systems at 10, 50, 100 Hz using 8^{th} order Butterworth filters.

In Figure 3 an example of a stress history for one channel is shown. As seen, the calculated and measured stresses correspond very well. The two time series have different length since the accelerations and strains are sampled on two separate systems and not stopped at the same time.

5. Experiment on Lattice Tower

5.1. Test Program for Lattice Tower

The vibration monitoring system, composed of six Shaevitz accelerometers and eight single 120 Ω strain gauges, was installed on the lattice tower for one month in the spring 2004. Three accelerometers were mounted at the top of the structure, three accelerometers near the middle of the structure and eight strain gauges were mounted near the support of the tower; six strain gauges on the legs 0.4 m above the support and two strain gauges on diagonals 1.3 m above the support. All strain gauges were measuring in the direction of the profiles.

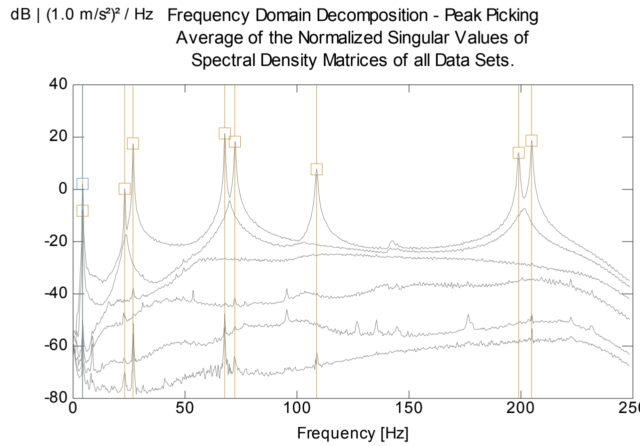

Figure 2 *Frequency Domain Decomposition of measurements on laboratory structure. Average of normalized singular values of spectral density matrices, 1024 frequency lines.*

Table 1 *Experimental and numerical determined natural frequencies, percentage deviation and MAC-values for pilot experiment.*

Φ [–]	f_{exp} [Hz]	f_{FE} [Hz]	Deviation [%]	MAC [–]
1	4.25	4.28	0.7	0.9941
2	4.25	4.29	0.9	0.9816
3	23.00	23.14	0.6	0.9917
4	26.75	27.00	0.9	0.9984
5	67.75	67.70	-0.1	0.9979
6	72.25	71.72	-0.7	0.9976
7	108.80	109.74	0.9	0.9915
8	199.00	195.66	-1.7	0.9846
9	204.80	202.12	-1.3	0.9962

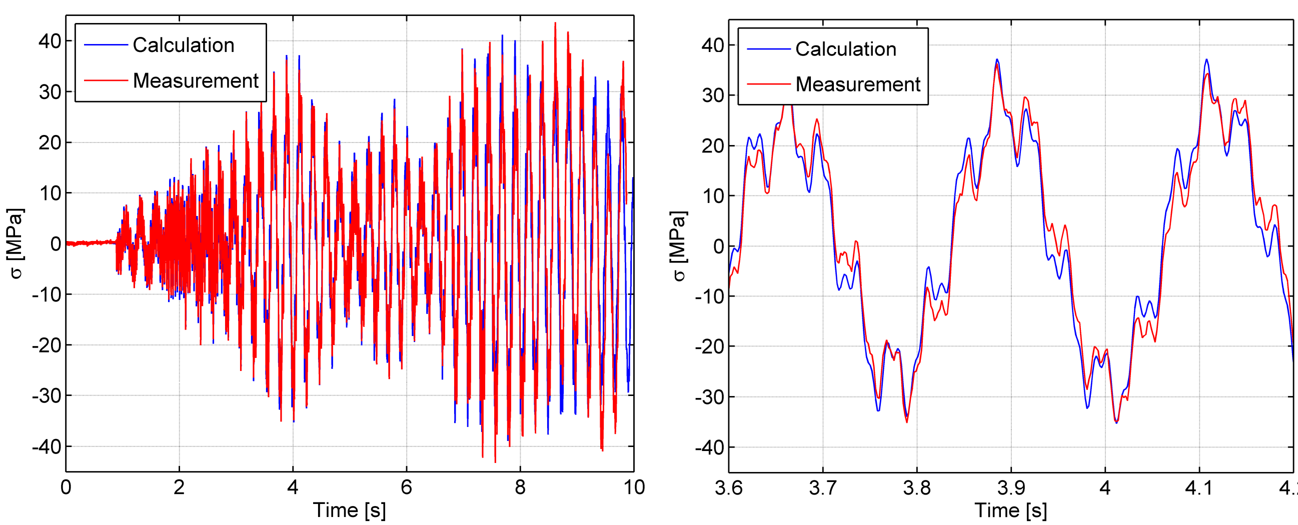

Figure 3 *Example of measured and calculated stress history in laboratory structure. (Left) Entire time series, (right) zoom in on 3.6–4.2 s.*

The monitoring system was initially setup to sample continuously one hour every forth hour, and during periods with high wind speed the monitoring was running continuously in several hours. In all, 223 hours of measurements were recorded. All measurements were sampled with 100 Hz.

The structure is modeled in the finite element program StaadPro using beam elements with six d.o.f. per node. The model is calibrated by applying springs in the support of the model.

5.2. Data Processing and Results of Experiments on Lattice Tower

Eight modes are identified from 30 minutes measurement series. In Figure 4 the results of the modal identification by FDD is plotted. In Table 2 the experimental and numerical determined natural frequencies are compared and MAC between $\mathbf{\Phi}_{FE}$ and $\mathbf{\Phi}_{exp}$ are calculated. In Figure 8 the mode shapes are plotted.

The modal identification is performed on different 30 minutes measurements series with different wind speeds and wind directions. By comparison the identified mode shapes from the different series, it is found that mode shape 1 and 2 are not stable. For this reason, the MAC between the experimental and numerical mode shape 1 and 2 are low, and the modal expansion must be done using the numerical mode shapes.

The numerical drift resulting from the integration is removed by a 5th order 0.2 Hz low-pass Butterworth filter.

Since the results are used in fatigue analysis it is important to include all the modes that contribute significantly to the fatigue damage. These modes are identified by calculating the damage vs. the number of modes. For instance, the modes of higher order are filtered out one by one (or in pairs) by low-pass filtering and for each filter step the accumulated damage is

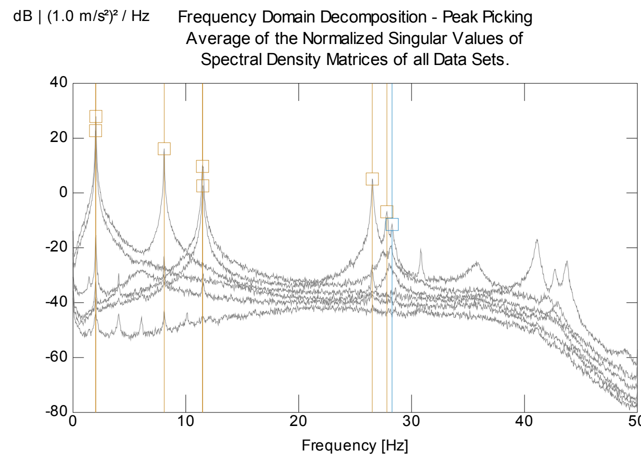

Figure 4 *Frequency Domain Decomposition of measurements on lattice tower. Average of normalized singular values of spectral density matrices, 2048 frequency lines.*

Table 2 *Experimental and numerical determined natural frequencies, percentage deviation and MAC-values for experiment on lattice tower.*

$\mathbf{\Phi}$ [–]	f_{exp} [Hz]	f_{FE} [Hz]	Deviation [%]	MAC [–]
1	2.00	2.01	0.5	0.7428
2	2.02	2.02	-0.3	0.7448
3	8.08	7.87	-2.5	0.9791
4	11.47	11.40	-0.6	0.9618
5	11.49	11.54	0.4	0.9671
6	26.51	25.75	-2.9	0.8289
7	27.80	29.49	6.1	0.8714
8	28.27	30.35	7.4	0.8520

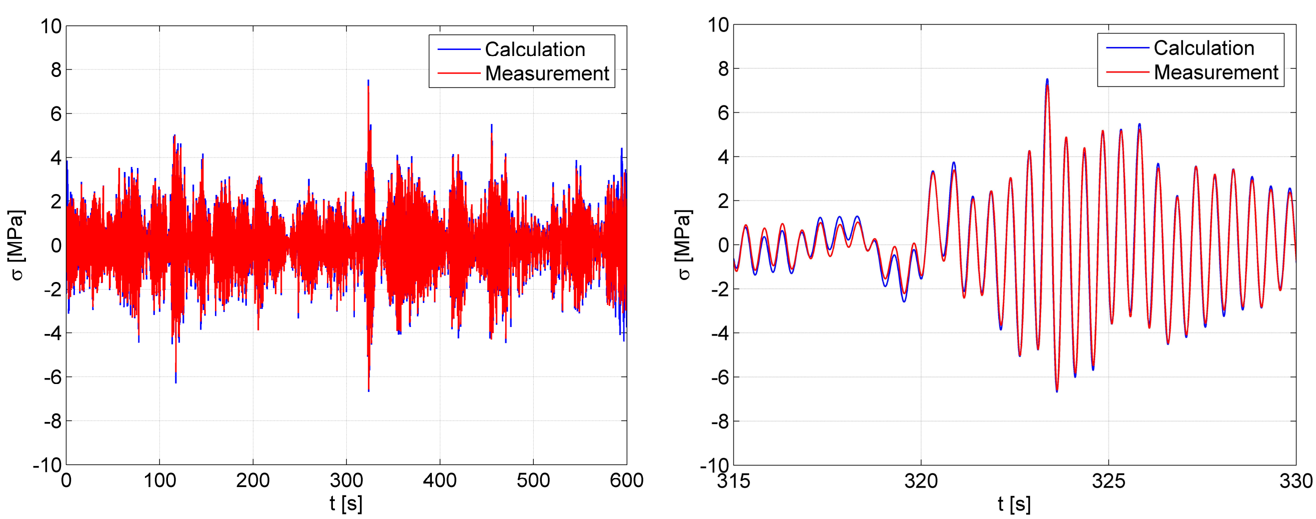

Figure 5 *Example of measured and calculated stress history. (Left) Entire time series, (right) zoom in on 315-330 s.*

calculated. If the damage is not reduced significantly by a low-pass filtering the filtered modes can be left out. In this manner it is found that only the two first modes of the lattice tower are participating significantly to the fatigue damage. Therefore, the measurements are run through an 8th order 5 Hz low-pass Butterworth filter and only mode shape 1 and 2 are used in the calculations.

In Figure 5 an example of a 10 min. stress history for channel 1 (leg) is shown, comparing the measured and calculated stresses. The figure shows that the stress histories have been calculated with great accuracy and it is verified that the modal expansion with use of the numerical mode shapes is applicable. In Graugaard-Jensen et al. [1] it is found that main part of the deviation between the measurements and calculations is caused by a small error on the estimated static response. This error can be minimized by expanding the static displacements in the complete set of mode shapes, thus including high order mode shapes.

Stress spectra are plotted and damage is calculated including all 223 hours of measurements. An example of such a stress spectrum is plotted for channel 1 (leg) in Figure 6, and in Table 3 the damage is listed for all eight channels. The damage is calculated with use of a linear SN-curve with the parameters log K = 16.786 and m = 5 and without any cut-off limit, cf. DS410 [7].

In inspection planning the uncertainty on the stress history is included in a Coefficient Of Variation (COV). For offshore structures COV is typically ranging from 0.10 to 0.15, cf. Faber et al. [7]. It is found that COV is reduced to 0.03 for the results of the lattice tower, illustrated in Figure 6 where COV is plotted by assuming that the uncertainty is modeled by a normal distributed 95% double-sided probability interval.

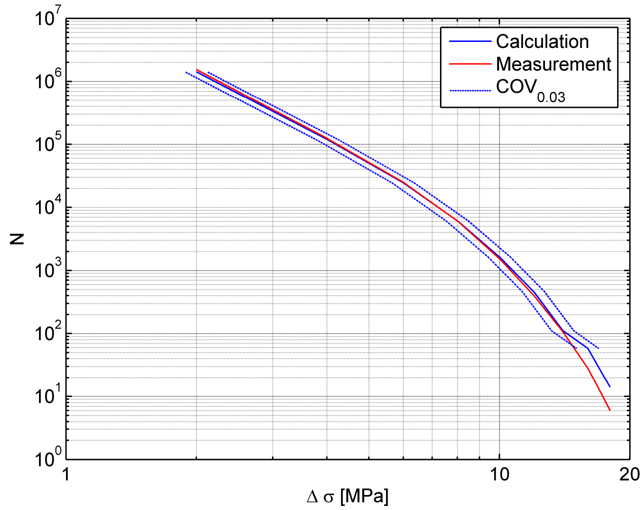

Figure 6 *Example of stress spectrum (channel 1, leg).*

Table 3 *Comparison of measured and calculated damage. The ± values indicates the 95% probability intervals for COV = 0.03.*

Ch. [–]	D_{meas} $1 \cdot 10^{-6}$	D_{calc} $1 \cdot 10^{-4}$
1	3.53	3.54+1.17/-0.93
2	3.55	3.43+1.14/-0.89
3	4.52	3.43+1.14/-0.89
4	0.88	0.73+0.24/-0.19
5	0.86	0.72+0.24/-0.19
6	3.38	3.54+1.17/-0.92
7	2.31	2.74+0.90/-0.72
8	2.22	2.22+0.74/-0.58

6. Conclusions

It has been shown that it is possible to calculate stress histories in any point of a structure with great accuracy by combining natural input modal analysis with finite element modeling. The advantage of the method is that the accelerations of the structure only have to be measured in few easily accessible points of the structure, and the stresses can be calculated in any arbitrary point of the structure. In this manner the calculated stress histories can replace strain gauge measurements in many cases and it allows for an accurate estimation of fatigue damage.

The coefficient of variation (COV) on the stress history is lowered to below 0.05. The result is that if the method is applied on e.g. offshore structures, the number of inspections can be reduced significantly since the number of these directly depends on the uncertainty on the stress history, cf. Graugaard-Jensen et at. [1].

References

[1] Graugaard-Jensen, J., Hjelm, H. P. and Munch, K., *Modal Based Fatigue Estimation*, M.Sc. Thesis, Aalborg University, Denmark, 2004

[2] Graugaard-Jensen, J., Hjelm, H. P. and Munch, K., *Fatigue Estimation by Natural Input Analysis*, www.svibs.com, 2004

[3] Van Overschee, P. and De Moor, B., *Subspace Identification for Linear Systems: Theory, Implementation Applications*, Kluwer Academic Publishers, 1996

[4] Brincker, R., Zhang, L. and Andersen, P., *Modal identification of output-only systems using frequency domain decomposition*, Smart Materials and Structures 10 (2001) pp. 441–445, 2001

[5] Kreyszig, E., *Advanced Engineering Mathematics*, John Wiley & Sons, 8th edition, 1998

[6] Cook, R. D., Malkus, D. S., Plesha, M. E. and Witt, R. J., *Concepts and Applications of Finite Element Analysis*, John Wiley & Sons, 4th edition, 2002

[7] DS410, *Standard for Load on Structures DS410*, Danish Standard, 4th edition, 1998

[8] Faber, M. H., Straub, D., Sørensen, J. D. and Tychsen, J., *Field Implementation of RBI for Jacket Structures*, OMAE'03, 2003

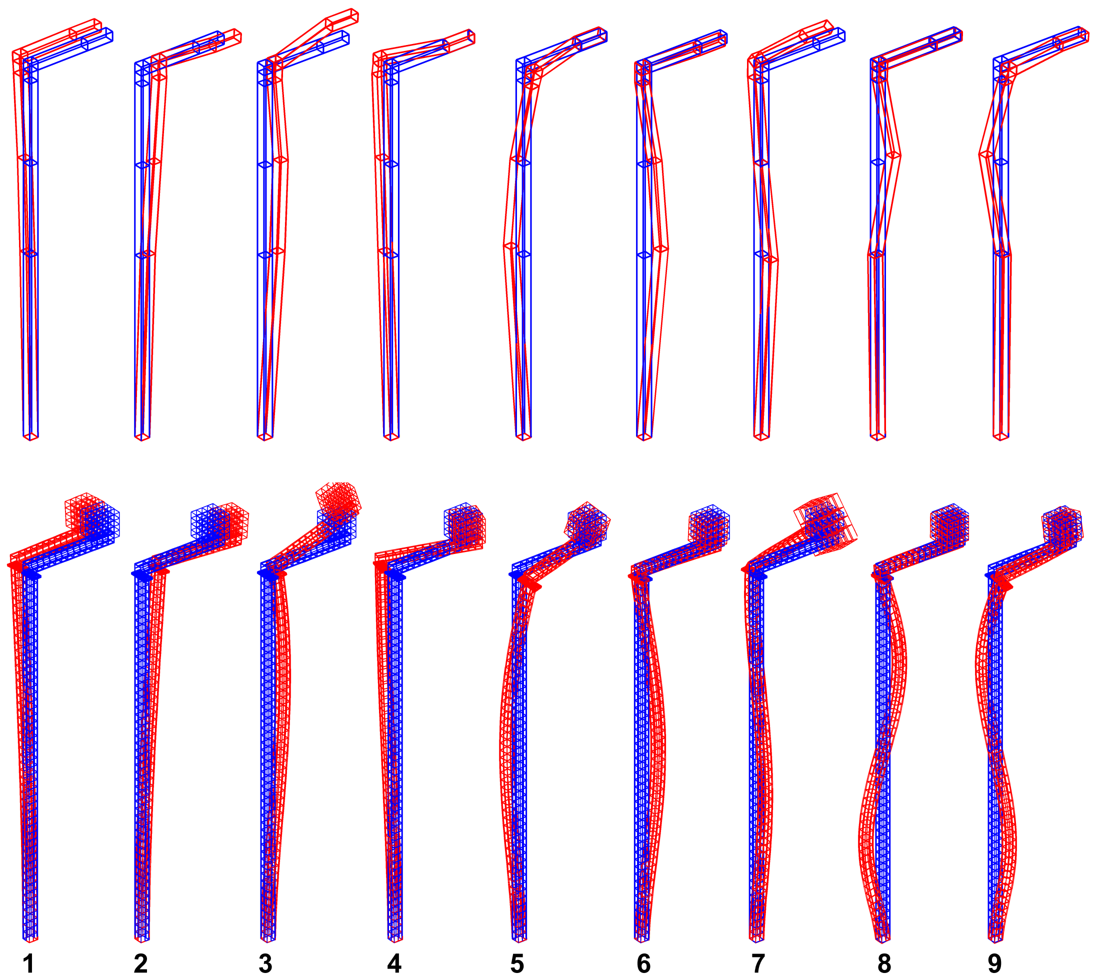

Figure 7 *The nine experimental (top) and numerical (bottom) mode shapes of the laboratory structure.*

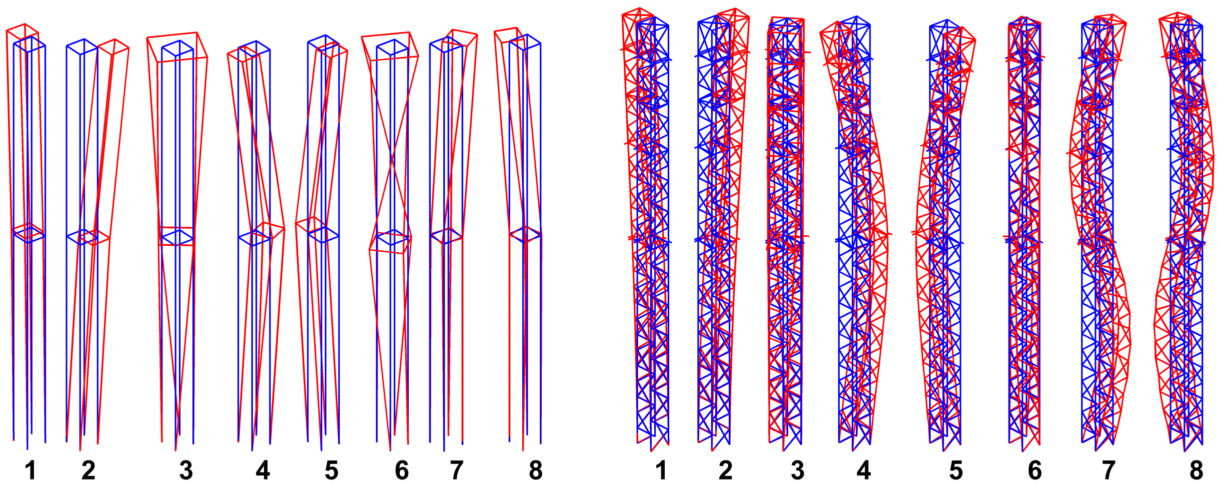

Figure 8 *The eight experimental (left) and numerical (right) mode shapes of the lattice tower.*