Operational Modal Analysis of the Braga Sports Stadium Suspended Roof

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1. ABSTRACT

This paper describes the application of three output-only modal identification techniques to the data collected during an ambient vibration test performed at the Braga Sports Stadium suspended roof. The identified natural frequencies and modes shapes are compared with the ones predicted by a finite element model that took into account the geometrical non-linear behavior and the construction process. On the other hand, the estimated modal damping coefficients are compared with the ones obtained in previously developed forced/free vibration tests. These comparisons allowed evaluating the accuracy of the results provided by the experimental and numerical tools used.

2. INTRODUCTION

The Braga Municipal Sports Stadium is the main infrastructure of the Braga North Urban Park, in the North of Portugal, that has been constructed recently and hosted some of the games of the 2004 European Football Championship. The stadium, designed by Eduardo Souto Moura in conjunction with the design office Afassociados, has been considered a masterpiece of Architecture.

The innovative characteristics of the roof structure have motivated extensive studies developed during the design phase by various independent entities, whose purpose was to adequately define the design wind load, evaluate the corresponding static and dynamic behaviour, and investigate the susceptibility to aeroelastic instabilities. These studies comprehended the development of different numerical models of the structure and a series of wind tunnel tests performed on physical models.

Figure 1 – The Braga Municipal Sports Stadium.

The Faculty of Engineering of the University of Porto was consulted in this context, at an early stage of the project, with the purpose of developing a static and dynamic study of the roof structure, which was used formerly in the definition of the geometric and mechanic characteristics of the cables and slabs, and whose dynamic properties were later used in the construction of a physical model for wind tunnel tests [1].

Forced and free vibration test were conducted at the end of construction, in order to identify the modal damping coefficients of the cable-roof, whose values are of great importance to analyze its vulnerability to wind excitation that might affect the structural durability.

Recently, a complete ambient vibration test of the suspended roof was developed by the authors, in order to identify the modal parameters of the structure (natural frequencies, mode shapes and modal damping coefficients). In this paper, the results provided by the application of three output-only identification techniques to collected data are compared with the natural frequencies and mode shapes of the developed numerical model and with the modal damping coefficients estimated from the forced and free vibration tests.

3. DESCRIPTION OF THE STRUCTURE

The stadium was constructed in the slopes of Monte Castro, and develops as an amphitheatre over a wide rural landscape, formed only by two rows of stands, on either side of the pitch, and by a granite massif (Figure 2). The most noticeable element of the stadium is its roof, which is formed by pairs of full locked coil cables with diameters varying between 86 and 80mm, spaced 3.75m apart from each other, supporting two concrete slabs over the two stands of the stadium. The cables' span is 202 m and the slabs length is 57.3 m, therefore the remaining 88.4 m of the central part are free. The rainwater is drained from the roof along one side only, the slope being achieved by a variation of the length of the cables. The concrete slabs have a thickness of 0.245m and are connected to the cables only in the normal direction, allowing relative tangential movements. A transversal triangular truss is suspended front the inner border of each concrete slab acting as a stiffness beam and simultaneously accommodating the floodlights and loud speakers.

The roof cables are anchored in two large beams at the top of both stands – east and west. The east stand is structurally made up of 50 m high concrete walls, whose geometry was defined in order to minimize the unbalance moments at the level of the foundation, motivated by the combination of the gravitational action of the stand and the high forces transmitted by the roof cables. In the west stand the concrete walls are anchored in the rock and the roof cables' tension forces are transmitted to the foundation by prestressing tendons embedded in the concrete.

The outstanding characteristics of the structure and the need of a tight control of the corresponding behaviour during the construction justified the installation of a monitoring system, which comprehends static and dynamic components. The static monitoring system was essential during the construction and is based on a series of load cells installed in the cables anchorages, internal instrumentation of the concrete structure (strain gauges, tiltmeters and thermometers), instrumentation of the rock massifs and foundations, load cells in the anchors to the earth and in-place inclinometers. The dynamic monitoring system is important to observe the response of the roof to the wind and is composed by 6 accelerometers, installed in the inner edges of the concrete slabs, and by cells to measure the wind pressure at various points on the underside and top of the roof covers.

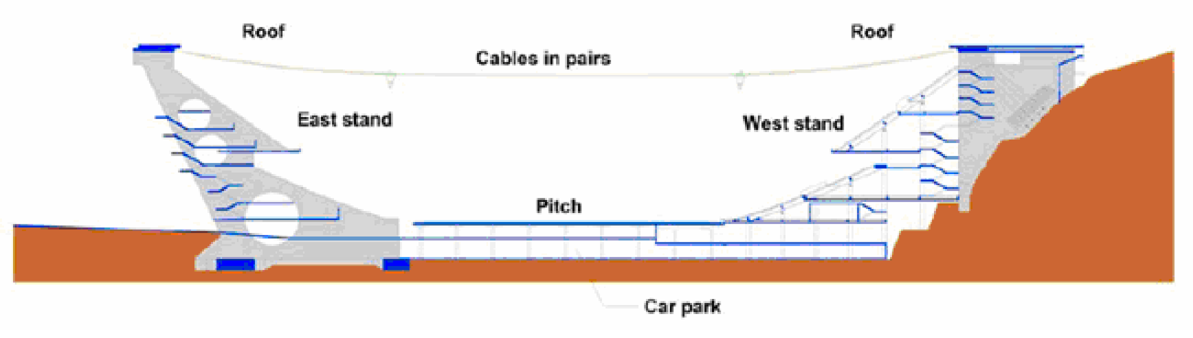


Figure 2 – Representation of the Braga Municipal Sports Stadium (picture extracted from http://www.afaconsultores.pt).

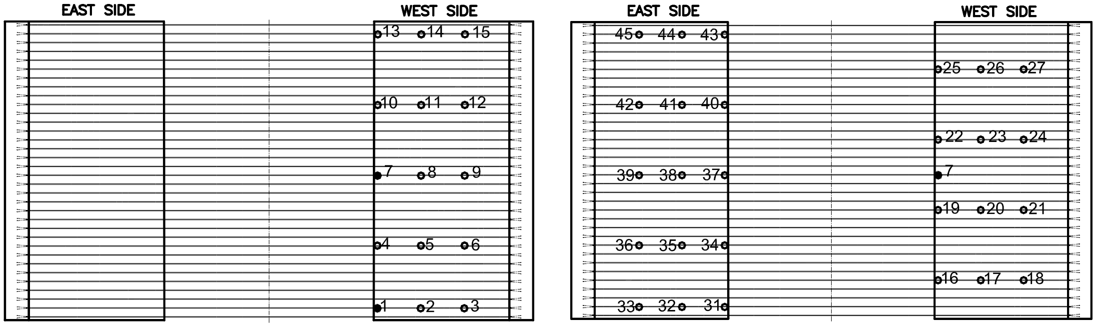
4. AMBIENT VIBRATION TEST DESCRIPTION

The ambient vibration test comprehended the measurement of the vertical acceleration at 42 points of the roof, using 3 strong motion recorders (Figure 3), synchronized by GPS and programmed using a laptop. The use of these recorders is very practical because no electrical cabling is required. In the test of this structure the need of cables connecting equipment placed in both sides of the roof, would make the test preparation much more complicated.

The test was performed in two days: on the first day the points of the west slab represented in Figure 3 a) were measured using 13 setups, while the points of the west and east slabs represented in Figure 3 b) were measured on the second day using 15 setups.

On the first day two reference points (points 1 and 7) were used. After a preliminary analysis of this data, it was concluded that for the frequency range of interest (0 - 1Hz) all the modes were detected by the reference sensor at point 7. So, on the second day, it was decided to use just this reference, in order to reduce the duration of the test. The second test was performed mainly to improve the geometric characterization of the mode shapes.

Figure 3 – Seismograph



b) Test 1

c) Test 2

● reference points ○ points measured with the moving sensors

Figure 4 – Points measured during the ambient vibration test:
a) 1st day; b) 2nd day.

For each setup, time series of 16 minutes were collected with a sampling frequency of 100 Hz (minimum frequency allowed by the acquisition system). Figure 5 represents one of the time segments colleted at point 7 (reference point). Figure 6 shows the variation of the standard deviation of the time segments measured at point 7 along the 28 setups. The amplitude of the vibrations is only dependent on the wind speed, as it was the only significant action on the roof during the test. So, the graphic shows that during the first day of tests (a rainy day) the wind speed was higher and had greater fluctuations than on the second day (a sunny day).

During the tests, the maximum value of recorded vertical acceleration was of 5 mg, denoting a very low level of oscillation of the roof structure.

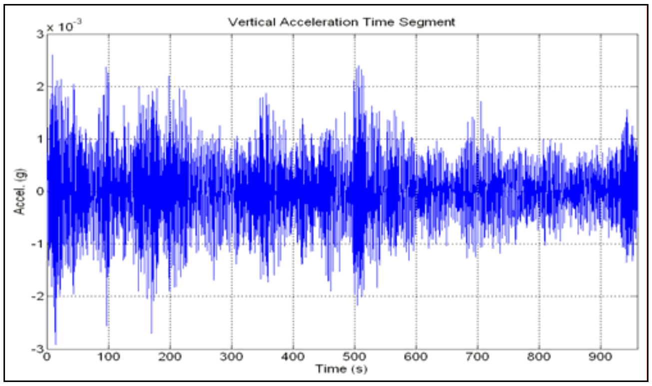


Figure 5 – Vertical acceleration time series measured at section 7 during the first setup.

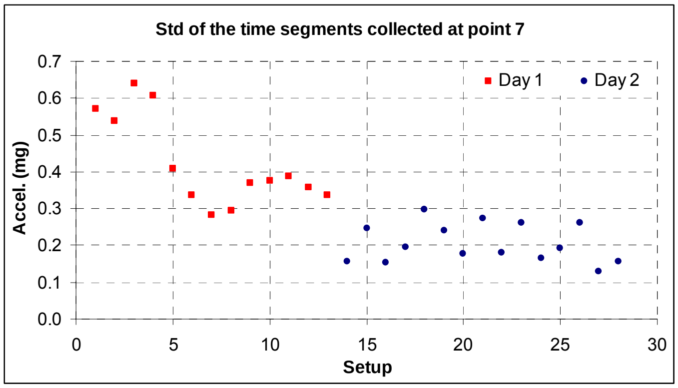


Figure 6 – Variation of the standard deviation of the time series measured at point 7 along the 28 setups.

5. IDENTIFICATION OF THE MODAL PARAMETERS

5.1 Frequency Domain Decomposition Method

5.1.1 Identification of natural frequencies and mode shapes

The ambient vibration data collected at the Braga Sport Stadium suspended roof during the two days of tests was first processed by the Artemis software using the Frequency Domain Decomposition method [2]. The Artemis input files were prepared in MatLab and a decimation of order 10 (to reduce the sampling frequency from 100 Hz to 10 Hz) was applied to minimize the processing effort. The spectra were calculated with a resolution of 0.00488 Hz, which allowed a good separation of the closely spaced modes and sufficient accuracy in the determination of the natural frequencies.

This first analysis was performed to identify the main frequencies of the structure and to estimate the mode shapes' configurations. Figure 7 represents the configurations of the first 6 identified mode shapes. Because of the existence of the slope in the roof to drain the rain water, the structure is not perfectly symmetrical in relation to the middle axis parallel to the cables. This particular geometry justifies the existence of pairs of modes with close frequencies and almost symmetrical configurations.

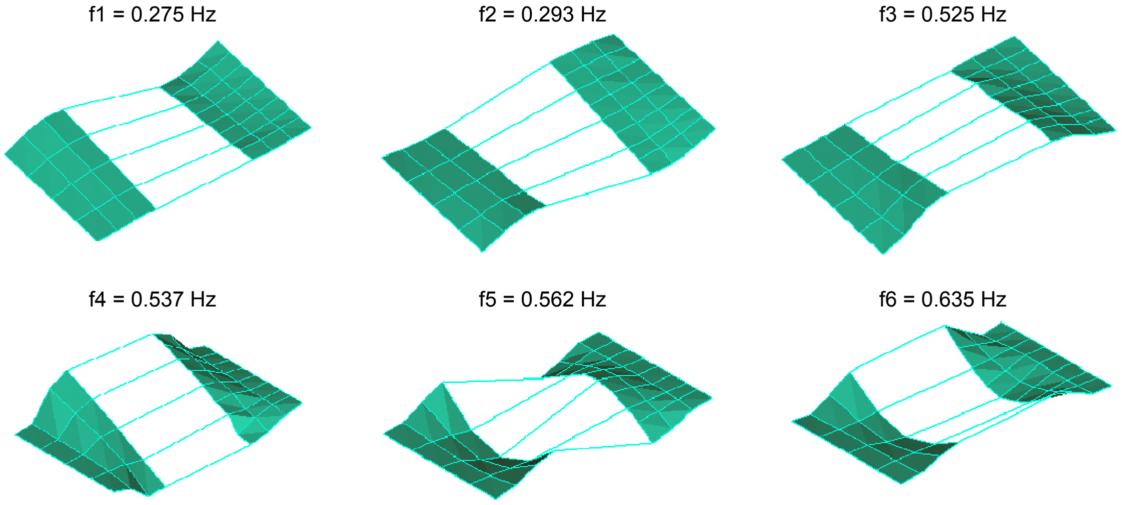


Figure 7 – Mode Shapes estimated by the FDD method.

5.1.2 Identification of modal damping coefficients

The modal damping coefficients can be estimated using an enhanced version of the FDD method, the EFDD method [3]. Usually, when the ambient vibration test is performed using several setups, the modal damping coefficients, as well as the natural frequencies, are estimated using independently the spectrum matrix of each setup and then, the final estimates are defined as the average of the estimates provided by all the setups.

However, in the test performed in this structure, the length of each setup time segments is not sufficiently long to accurately estimate modal damping coefficients of modes with so low natural frequencies, especially because very small values of damping are expected.

On the other hand, on the first day of tests the vertical accelerations at points 1 and 7 (points defined as references in Figure 3 a) were measured simultaneously during the 13 setups. These 13 time series with 16 minutes are a good basis to evaluate the modal damping coefficients.

Beyond that, to increase the accuracy of the estimates, the auto-correlation functions were calculated using an alternative procedure to the presented in [3], in order to avoid the circular error [4]. These were obtained using an adaptation of the procedure described in [5], which is also based on a FFT approach and comprehends the following steps: selection of time segments from the measured signals without application of windows, duplication of the length of the time segments by adding zeros at the end, calculation of auto-spectra and cross-spectra functions to construct the spectrum matrix, averaging of the spectra estimates to reduce the random error, singular value decomposition of the spectrum matrix, selection of the singular values associated to a selected mode (using the MAC values), inverse FFT of the spectrum formed by the selected points and zeros in the other abscises and, finally, division of the resulting autocorrelation function by a triangular window (w(t)=(T-t)/T, where T is the length of the selected time segments). The modal damping coefficient of the mode under analysis is then estimated from the exponent of an exponential function adjusted to the relative maxima of the autocorrelation function.

In the processing of the available data collected at points 1 and 7 time segments with 11 minutes were selected using an overlap of 54%, allowing the accomplishment of 26 averages. The singular values calculated from the spectrum matrix, with dimension 2x2, are represented in Figure 8. In the same graphic the points chosen to estimate the auto-spectra (Spi) associated with 6 different modes are selected (the points with MAC > 0.8). For the other modes, identified in the graphic by the peaks, it is not possible to select well defined auto-spectra, because of the proximity between resonance peaks. The natural frequencies identified in the graphic are not exactly the same as those presented in Figure 7, because in this analysis the frequency resolution is higher (0.00153 Hz).

Two of the auto-correlation functions calculated from the identified auto-spectra (using the inverse FFT) are represented in Figure 9, to show the type of graphics used to fit the exponential free decays and estimate the modal damping coefficients presented in Table I.

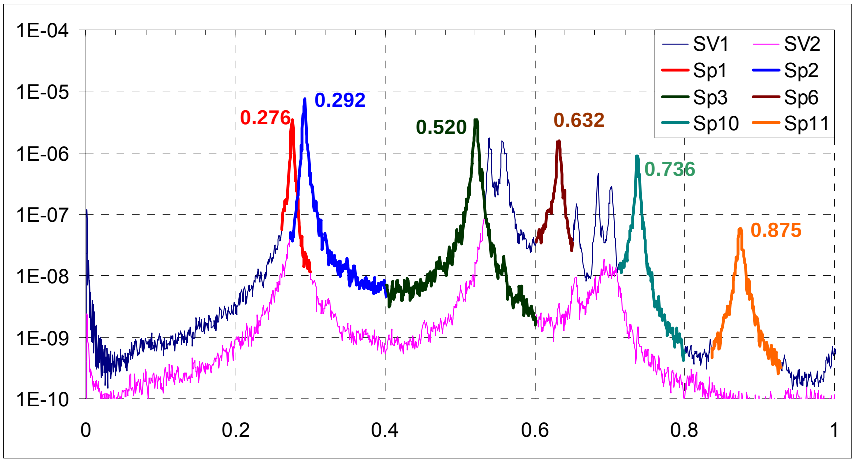


Figure 8 – Singular values of the spectrum matrix calculated using the time segments collected at the reference points (1 and 7) and selection of the auto-spectra of each mode.

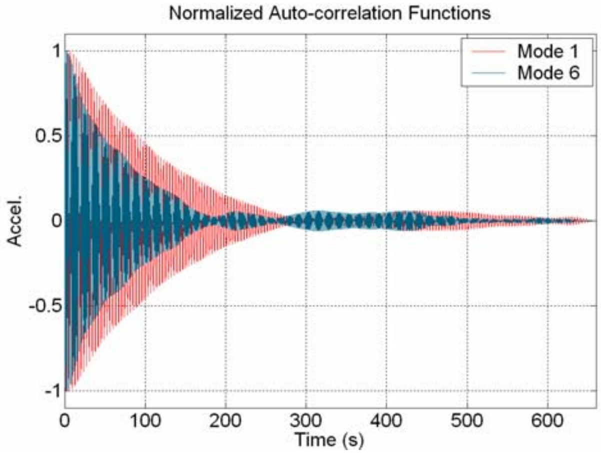


Figure 9 – Normalized auto-correlation functions associated with

Table I – Natural frequencies and modal damping coefficients (EFDD method).

Mode	f (Hz)	damp. (%)
1	0.276	**0.58**
2	0.292	**0.52**
3	0.523	**0.47**
4	0.539	**-**
5	0.556	**-**
6	0.632	**0.35**
7	0.655	**-**
8	0.684	**-**
9	0.703	**-**
10	0.737	**0.25**
11	0.874	**0.36**

modes 1 and 6.

5.2 *Stochastic Subspace Identification Methods*

The mode shapes identified by the FDD method are of very good quality and the accuracy of the SSI methods to identify mode shapes is already proven. So, the application of the stochastic subspace identification methods was performed mainly to get estimates of the modal damping coefficients. For this purpose, the data collected on the first day of tests was used, as for the application of the enhanced version of the FDD method, because it has the advantage of having two reference points.

5.2.1 Covariance driven Stochastic Subspace Identification Method (SSI-COV)

The Covariance driven Stochastic Subspace Identification Method was applied using routines developed in MatLab at the University of Porto [6]. A description of this method and of its theoretical background can be found in reference [7].

Before the application of the identification algorithm, the measured signals were filtered by a low pass filter with a cut-off frequency of 1 Hz and a decimation of order 20 was applied (reducing the sampling frequency from 100 Hz to 5 Hz).

The correlation functions are the basis of this identification method. These can be estimated using three different procedures: using directly the definition that involves the realization of a sum, using the FFT or using the Random Decrement. In the present application, the first alternative was used, which is not very computationally efficient, but does not introduces bias and is easy to programme.

Furthermore, the number of points of the correlation functions used by the identification method has to be defined. This parameter, usually represented by 2.*i*, has influence on two aspects: the maximum order of the estimated model is equal to *i.r* (*r* being the number of points measured in each setup); the quality of the stabilization diagrams and consequently of the estimated modal parameters. This last aspect is illustrated in Figures 10 and 11, which show that increasing the *i* parameter, the quality of the stabilization diagrams also increases (better defined vertical alignments of stable poles). The increase of *i* has also the drawback of increasing substantially the calculation effort, so it is important to find a balance between the desired quality and the needed processing time. Taking that into account, in the present application was adopted *i = 100*, meaning that the segments of the correlation function selected for the identification contain 2x100/5x0.275 = 11 cycles of the lowest frequency. It is important to state that, if the decimation was not applied, it would be necessary to adopt a higher *i* to have the same number of cycles and consequently, increase dramatically the calculation effort.

The data of the 13 setups was processed independently using the three collected time segments and so, 13 stabilization diagrams, like the one presented in Figure 11, were constructed. The presented diagram shows that the dynamic behaviour of the structure is well represented by state-space models of order between 20 and 40. The use of models of this relatively small order was only possible because a low pass filter was applied, that eliminated the contribution of the modes with frequencies greater than 1 Hz.

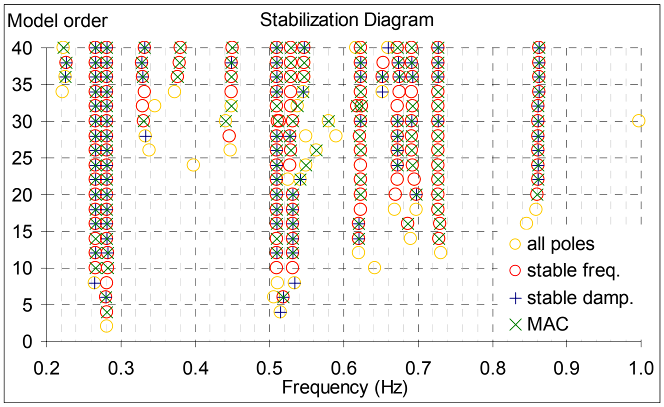


Figure 10 – Stabilization diagram of setup 4

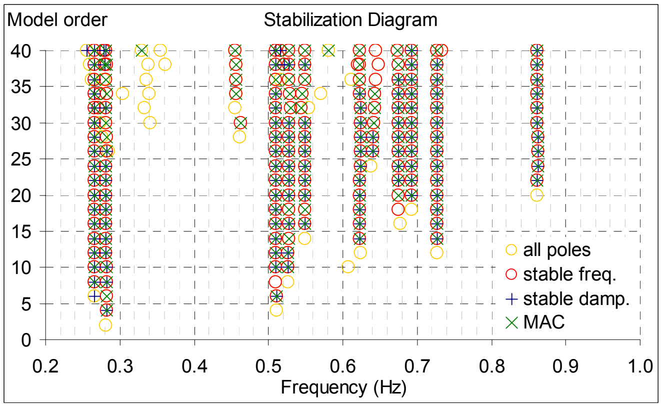


Figure 11 – Stabilization diagram of setup 4

with *i* = 50. with *i* = 100.

In the 13 stabilization diagrams, the order of the most suitable model was selected and the natural frequencies and the modal damping coefficients of that model were calculated. Figure 12 shows the 13 estimates of frequencies and modal damping coefficients for the 11 identified modes. The graphic shows that the variation of the natural frequencies along the setups is very small whereas the modal damping coefficients have a significant dispersion. This dispersion shows the difficulty of getting reliable estimates of this parameter, which is justified not only by the uncertainties of the method but also by the variation of the damping with the levels of oscillation associated with the wind speed. Table II presents the mean and the standard deviation (Std) of the identified modal parameters.

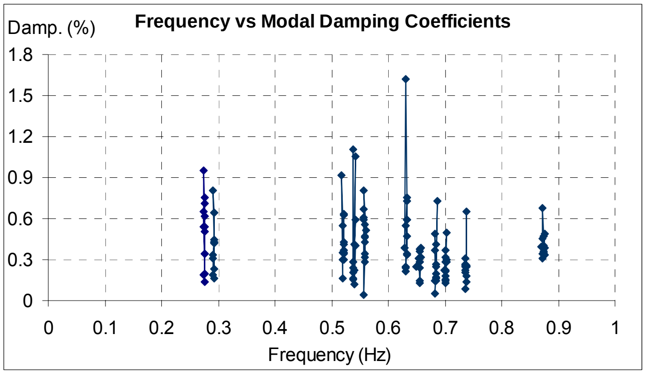


Figure 12 – Dispersion of the estimated modal damping coefficients.

Table II – Natural frequencies and modal damping coefficients (SSI- method).

Mode	f. (Hz)	Std. f. (Hz)	damp. (%)	Std. Damp. (%)
1	0.276	0.0008	0.50	0.25
2	0.292	0.0007	0.42	0.20
3	0.521	0.0014	0.44	0.20
4	0.539	0.0014	0.40	0.33
5	0.558	0.0012	0.47	0.19
6	0.632	0.0012	0.54	0.37
7	0.655	0.0019	0.28	0.09
8	0.684	0.0010	0.27	0.19
9	0.702	0.0010	0.26	0.10
10	0.737	0.0009	0.26	0.13
11	0.874	0.0016	0.41	0.10

5.2.2 Data driven Stochastic Subspace Identification Method (SSI-DATA)

Complementary, the data driven stochastic subspace identification method was applied with the Artemis software and using the data provided by the pre-processing procedures described in the previous section (decimation and filtering). In Artemis software the user has to define the expected number of modes, including the structural modes, the modes associated with harmonic loads, and the modes needed to model the noise. This parameter is an indirect way of defining the value of *i* – one half of the number of rows of the Hankel matrix [8], which is closely related with the *i* parameter of the SSI-COV method. In this application a very conservative number of modes was defined (a total of 100), allowing to estimate state space models with orders up to 200.

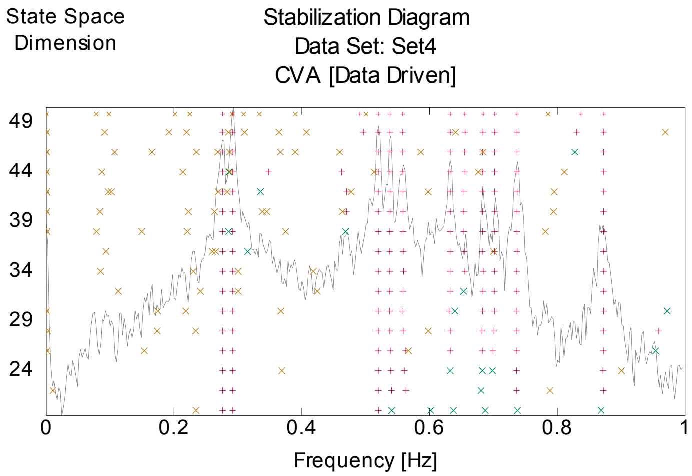


Figure 13 – Stabilization Diagram of setup 4

Table III – Natural frequencies and modal damping coefficients (SSI-DATA method).

Mode	f. (Hz)	Std. f. (Hz)	damp. (%)	Std. Damp. (%)
1	0.275	0.0007	0.51	0.20
2	0.292	0.0005	0.48	0.22
3	0.521	0.0008	0.39	0.09
4	0.539	0.0027	0.33	0.22
5	0.558	0.0012	0.53	0.23
6	0.631	0.0012	0.47	0.25
7	0.654	0.0032	0.73	0.51
8	0.684	0.0012	0.30	0.15
9	0.701	0.0010	0.31	0.17
10	0.737	0.0010	0.29	0.14
11	0.875	0.0015	0.38	0.10

The inspection of the stabilization diagrams, like the one represented in Figure 13, showed that models with orders between 30 and 50 are able to characterise the dynamic behaviour of the structure. Anyway, as in the SSI-COV method, the over estimation of the *i* parameter is important to improve the quality of the stabilization diagrams and of the estimated modal parameters. Table III summarizes the results (mean and standard deviation of the frequencies and modal damping coefficients estimated from the data of the 13 setups) and shows, once again, the dispersion of the estimates of the modal damping coefficients. Despite of the observed dispersion, the estimates provided by all the applied methods, and for the majority of the modes, are similar.

6. RESULTS DISCUSSION

6.1 Correlation between the experimental and numerical frequencies and mode shapes

At an early stage of roof structure's design the University of Porto developed a numerical model used to perform a parametric study that gave support to the definition of the concrete slab geometry and of the cables cross section [1]. This model took into account the geometrical non-linear behavior and the construction process that comprehended the following main steps: installation of the cables, assembling of prefabricated panels (1.8 m x 3.75 m) over the cables and cast of the transversal and longitudinal joins between the panels.

Subsequently, the numerical model was updated to take into account some adjustments of the structure characteristics dictated by the construction process. Figure 14 represents the first six numerical mode shapes of the updated model, which are in good agreement with the experimental ones (Figure 7), as is revealed by the MAC matrix represented at Figure 15. In Table IV the natural frequencies calculated in the design phase and after construction are compared with the experimental ones, showing that, the last model characterizes very well the real dynamic characteristics of the roof. These results also prove that the identification techniques provided accurate results in term of mode shapes and natural frequencies, which were almost coincident using the three applied techniques.

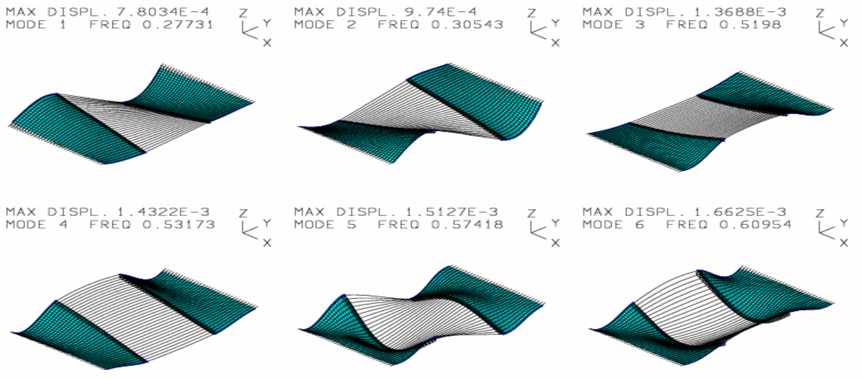


Figure 14 – Numerical Mode Shapes.

Table IV – Natural frequencies: calculated at design, after construction, and identified (SSI-COV).

Mode	**Frequency (Hz)**		
	Design	**Constructed**	**Identified**
1	0.303	0.277	0.276
2	0.322	0.305	0.292
3	0.455	0.520	0.521
4	0.470	0.532	0.539
5	0.476	0.574	0.558
6	0.516	0.610	0.632
7	0.660	0.673	0.655
8	0.672	0.678	0.684
9	0.691	0.712	0.702
10	0.693	0.754	0.737
11	0.712	0.844	0.874

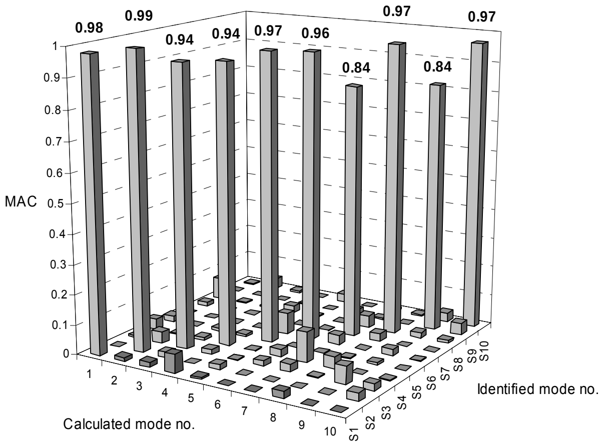


Figure 15 – MAC matrix for identified vs calculated vibration modes.

6.2 Validation of the estimated modal damping coefficients

The identification of modal damping ratios on the prototype structure was developed, in a first instance, at the request of design office Afassociados [9], based on a set of data collected by the instrumentation installed at the roof structure, during the forced and free vibration tests developed at commissioning phase. In this section, the estimated modal damping coefficients are compared with the ones provided by the output-only identification techniques.

The free vibration test was based on the sudden release of a 5 ton mass from a point located close to mark 1 in Figure 4. The response was collected by the six tri-axial force-balance accelerometers of the dynamic monitoring system, which are located at marks 1, 7, 13, 31, 37 and 43 of Figure 4. Figure 16 represents the response measured at point 31. The application of band-pass filters to the measured signals enables the evaluation of modal free decay responses; these were used to estimate the modal damping coefficients presented in the first column of Table IV. This procedure faces two problems, which justify the missing values in the table: the low level of excitation of some modes and the difficulty of isolate the contribution of modes with very close natural frequencies. An alternative to this procedure is to use the measured response to the impulse as input to the SSI-COV method, as the responses to impulses are proportional to the correlations of the responses to a white noise excitation. This technique was used after the application of a low-pass filter, with a cut-off frequency of 1 Hz, and a decimation to reduce the sampling frequency to 5 Hz, and provided the results presented in the second column of Table IV. It is interesting to observe that for the modes where both techniques were applied the results are very consistent.

Forced vibration tests were further conducted, based on a harmonic excitation of the roof at resonance, by means of a cable pulled by an electric engine from marks 1 or 7 (Fig. 4). After resonance was attained, the excitation was suppressed and the free vibration response measured at the same 6 accelerometers. Figure 17 represents one of the measured free decays. Using this procedure 5 modes were excited and so, 5 free decays were measured, like the one represented in Figure 17, that were used to estimate the modal damping coefficients presented in the second column of Table IV.

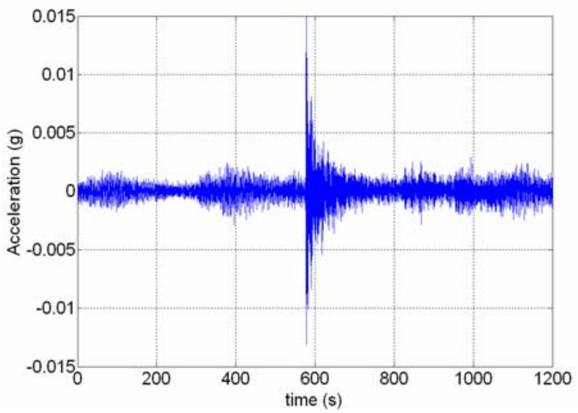


Figure 16 – Free decay after application of an impulse

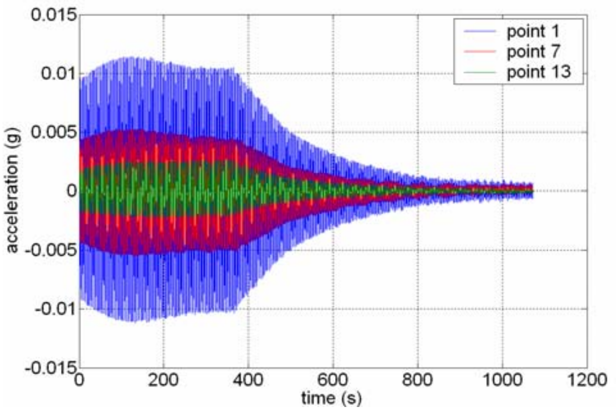


Figure 17 – Free decay after excitation of the 2nd mode

The comparison between the modal damping coefficients identified using artificial and ambient excitation shows the existence of a satisfactory correlation. However, one can notice that relative differences tend to increase at lower frequencies. In effect, it is well known that it is very difficult to estimate modal damping coefficients since they are dependent on the amplitude of vibration and also on the wind characteristics because of the existence of the aeroelastic damping. The very low damping values of this structure make the comparison even more difficult because very small differences are expressed by significant relative errors.

It is important to stress that in this very flexible structure the results provided by the FDD method are comparable with the ones of the SSI methods because very long time series were used and because it was adopted an alternative procedure to estimate the correlation function. The application of the standard EFDD method using independently the time series of each setup (with 16 min.) led to values of modal damping coefficients for the fist modes of about 1%.

Table IV – Summary of all the identified modal damping coefficients (%).

Mode	**Free Vib. Filter**	**Free Vib. SSI-COV**	**Harmonic Excitation**	**Ambient Vibration**		
				FDD	**SSI-COV**	**SSI-DATA**
1	-	0.29	0.28	0.58	0.50	0.51
2	-	0.37	0.27	0.52	0.42	0.48
3	0.28	0.32	0.22	0.47	0.44	0.39
4	0.25	0.22	-	-	0.40	0.33
5	-	0.44	-	-	0.47	0.53
6	0.34	0.36	0.43	0.35	0.54	0.47
7	-	0.29	-	-	0.28	0.73
8	-	0.11	0.20	-	0.27	0.30
9	-	0.18	-	-	0.26	0.31
10	0.20	0.18	-	0.25	0.26	0.29
11	-	-	-	0.36	0.41	0.38

7. CONCLUSIONS

The paper describes the ambient vibration test conducted in an innovative structure and presents in detail some alternative techniques to estimate its modal parameters using the structure's responses to ambient actions. It is shown that the available techniques can provide very accurate estimates of natural frequencies and mode shapes, the obtained estimates being very coherent and well correlated with the results provided by the developed numerical model.

Special action was devoted to the problematic task of modal damping coefficients estimation. It is shown that even in this challenging structure, with extremely low natural frequencies and damping factors and with closely spaced modes, the operational modal analysis can extract the order of magnitude of these parameters (absolute differences less than 0.25 %), constituting therefore an alternative to the more costly, but certainly more accurate, procedures based in artificial excitation. Further research is considered needed to explore the possibility of improving the capacity of operational modal analysis to estimate modal damping coefficients.

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