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SEISMIC DAMAGE ASSESSMENT IN STRUCTURES USING STOCHASTIC SUBSPACE-BASED ALGORITHM

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Abstract. *The problem of assessing the structure state following potential damage occurrence by exploiting vibration signal measurements produced by earthquake excitation is addressed. A damage assessment algorithm for structures under earthquake excitation is introduced. This method is based on a residual associated with output-only subspace-based modal identification and global* χ^2 *-tests built on that residual. This method makes effective use of the non-stationary and limited duration earthquake excitation, handles its stochastic uncertainties, and further assesses the structural damage. Since records have limited size, a new computation scheme, based on the stochastic bootstrap, is used to compute the residual covariance. An instrumented bridge, the Painter Street Overpass in California that has experienced more than ten earthquakes of different intensities was selected as a case study. Numerical results obtained from these experimental data are reported, using both identification and detection methods.*

1 INTRODUCTION

During the last three decades, vibration-based methods have been proven useful for the detection and localization of structural damage, which are current problems in Structural Health Monitoring field[1-4]. A variety of vibration-based damage detection methods have been proposed. Farrar et al.[5] provided an extensive review on this subject. Most of these methods are only suitable for the structures serving normally but not in catastrophic events, such as earthquakes and hurricanes. The rapid assessment after catastrophic events of critical structures like bridges, power stations, dams and gymnasia, is mandatory for the concerned government agencies. Therefore, there is a considerable demand for assessing the damage under earthquake excitation, particularly for large critical structures. In the earthquake excitation cases, there is no easy way to understand the detail of the excitation accurately, therefore only those methods based on output-only vibration are relevant. Since seismic waves are initiated by irregular ground motion, which is a complex non-stationary stochastic process, the conventional approaches based on Fourier transform can hardly deal with this kind of signals. Moreover, the following two characteristics of earthquake excitation are important challenges: 1) the transient nature, it means the total duration of earthquake is usually dozens of seconds, which result in a very limited output data record length; 2) the strong non-stationarity: both the possible non-linear dynamic characteristics of structures and the strong stochastic ground motion lead to the non-stationary characteristics of the structural dynamic responses.

The objectives of this work are to assess the structure state following potential damage occurrence by exploiting vibration measurements produced by earthquake excitation and discuss the feasibility of the method used in the analysis. This paper is divided into two parts. First, natural frequencies and mode shapes are derived for the structure by system identification analysis. The natural frequencies and the mode shapes of the damaged bridge were extracted by ARTeMIS Extractor program for the earthquakes. The Frequency Domain Decomposition-Peak Picking method was used for the analyses, so no value for damping coefficient was determined. Second, a damage assessment algorithm for structures under earthquake excitation is introduced. This method is based on a residual associated with output-only subspace-based modal identification and global χ^2-tests[6] built on that residual. This method can make use of the non-stationary and limited duration earthquake excitation effectively, handle its stochastic uncertainties, and further assess the structural damage. A new computation scheme for the residual covariance has been introduced: since records have limited sample size, stochastic bootstrap is used. All these steps are collected and implemented in the damage diagnosis software COSMAD.

An instrumented bridge, the Painter Street Overpass, in California, USA that has experienced more than ten earthquakes of different intensities was selected as a case study. The system identification method and the damage assessment method were implemented to assess the status of the bridge for selected earthquakes. In the study, one earthquake record is the reference record and all the other earthquake records are compared with it.

The remainder of this paper is organized as follows. The modeling and the identification technique is presented in section 2. A subspace-based modal detection algorithm and the special algorithm for processing seismic signals are presented in section 3. In section 4, the example, which includes the description of the bridge and earthquake records, and then modal identification and damage detection results, is presented in detail. Some conclusions are drawn in the last section.

2 IDENTIFICATION

Monitoring of structures under operational vibration leads to the discrete time model :

$$\begin{cases} X_{t+1} = FX_t + V_{t+1} \\ Y_t = HX_t \end{cases} \tag{1}$$

The modal parameters are found from both H and the eigenstructure $(\lambda, \varphi_\lambda)$ of the state transition matrix F in (1):

$$\psi_\lambda = H\varphi_\lambda \tag{2}$$

The collection of modes $(\lambda,\ \psi_\lambda)$ in (2) is also considered as the system parameter θ:

$$\theta = \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix} \tag{3}$$

where Λ is the vector whose elements are the eigenvalues λ, Φ is the matrix whose columns are the mode shapes, and **vec** is the column stacking operator.

The system identification of the Painter Street Overpass Bridge presented in the example was conducted using a non-parametric method called Frequency Domain Decomposition (FDD) implemented in the Operational Modal Analysis software called ARTeMIS Extractor. This system identification technique gives fast and reliable results even in the presence of noise and limited number of samples. It is a peak picking technique, where the modes are estimated by picking the peaks of the singular values of the estimated spectral densities.

The principle in the FDD technique is easiest illustrated by realizing that the measurements, collected in vector Y, can by written in modal coordinates

$$Y_t = \psi_1 q_1(t) + \psi_2 q_2(t) + \ldots = \Phi\ Q_t \tag{4}$$

where Φ is the matrix collecting all the mode shapes ψ_i as column vectors, Q_t is the vector of the corresponding modal coordinates $q_i(t)$.

Now obtaining the covariance matrix of the responses

$$\mathbf{C}_{YY}(\tau) = E\left\{ Y_{t+\tau} Y_t^T \right\} \tag{5}$$

and using equation (4) leads to

$$\mathbf{C}_{YY}(\tau) = E\left\{ \Phi Q_{t+\tau} Q_t^H \Phi^H \right\} = \Phi\, \mathbf{C}_{QQ}(\tau) \Phi^H \tag{6}$$

Expressing that the covariance of the measurements is related to the covariance of the modal coordinates through the mode shape matrix. The ^{H} is the Hermitian transposed operator. The equivalent relation in frequency domain is obtained by taking the Fourier transform

$$\mathbf{G}_{YY}(f) = \Phi\ \mathbf{G}_{QQ}(f) \Phi^H \tag{7}$$

Thus if the modal coordinates are uncorrelated, the spectral density matrix $G_{QQ}(f)$ of the modal co-ordinates is diagonal, and thus, if the mode shapes are orthogonal, then equation (7) is a Singular Value Decomposition (SVD) of the response spectral density matrix. Therefore, FDD is based on taking the SVD of the spectral density matrix

$$\mathbf{G}_{YY}(f) = \mathbf{U}(f) S_V(f) \mathbf{U}(f)^H \tag{8}$$

The matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \cdots]$ in (8) is a matrix of orthonormal singular vectors and the matrix $S_V = diag[s_i]$ is a diagonal matrix of corresponding singular values. This singular value decomposition is performed at all frequencies of the estimated spectral densities, making the decomposition dependent on the frequency *f*. As it appears from this explanation, plotting the singular values of the spectral density matrix will provide an overlaid plot of the auto spectral densities of the modal coordinates. A mode is identified by looking at where the line drawn from the first singular value has a peak. In the following it is assumed that a peak is found at frequency f_0. This frequency defines the natural frequency estimate of the mode. The corresponding mode shape is obtained as the corresponding first singular vector $\mathbf{u}_1$ in $\mathbf{U}$.

$$\psi = \mathbf{u}_1(f_0) \tag{9}$$

By searching through all the peaks of the singular value diagram the natural frequency and the mode shapes of all the structural modes can be estimated in a fast way. Since this technique is completely non-parametric it is a robust way to identify the modal parameters even in the case of limited samples. The limitations of the method are that the natural frequency estimates are dependent upon the frequency resolution of the spectral densities, and in its basic form presented here there is no damping estimation provided. It is possible to overcome these limitations using the so-called Enhanced Frequency Domain Decomposition (EFDD), see Brincker et al. [12].

3 SUBSPACE-BASED DAMAGE DETECTION

3.1 Description of the damage detection algorithm

In the proposed damage detection method is stated as the problem of detecting changes in the canonical parameter vector θ, defined in (3). It is assumed that a reference value θ_0, which generally is identified using recorded data on the undamaged system, is available. Given, on one hand, a reference value θ_0 of the model parameter and, on the other hand, a new data sample, the detection problem is to decide whether the new data are still well described by this parameter value or not. This approach basically addresses the early warning of small deviations of the system parameter. The key idea is to define a convenient *residual*, and to compute both the sensitivity of the residual with respect to damages and the uncertainty in the residual due to process noise and estimation errors.

The system eigenstructure can be monitored through an empirical Hankel matrix $\hat{H}_{p+1,q}$, as described in ref. [7]. The choice of the residual function associated with the parameter vector θ in (3) comes from the following remark. Assume that eigenvectors of matrix F are chosen as a basis for the state space of model (1). In this basis, the observability matrix is written as in [7]:

$$O_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix} \tag{10}$$

where diagonal matrix Δ is defined as $\Delta = \mathrm{diag}(\Lambda)$, and Λ and Φ are as in (3).

Whether a nominal parameter θ_0 is in agreement with a given output covariance sequence i s characterized by:

$$O_{p+1}(\theta_0) \text{ and } H_{p+1,q} \text{ have the same left kernel space.} \tag{11}$$

This property can be checked as follows. From the nominal modal parameter vector θ_0, compute $O_{p+1}(\theta_0)$ using (10), and pick any orthonormal matrix *S* such that

$$S^T O_{p+1}(\theta_0) = 0 \tag{12}$$

Matrix *S* in (12) depends implicitly on parameter θ and can be treated as a function of θ_0, denoted by $S(\theta_0)$. The parameter θ_0 corresponding to the covariance sequence is characterized by:

$$S^T(\theta_0) H_{p+1,q} = 0 \tag{13}$$

Assume now that a reference parameter θ_0 and a new data sample $Y_1, \ldots, Y_n$ are available. For checking whether the data agree with θ_0, the idea is to compute the empirical covariance sequence and fill in the empirical block-Hankel matrix $\hat{H}_{p+1,q}$ from the new data and to define the residual:

$$\zeta_n(\theta_0) = \sqrt{n}\,\mathrm{vec}(S^T(\theta_0)\hat{H}_{p+1,q}) \tag{14}$$

where *n* is the size of the measured data set. Let θ be the actual value of the parameter for the system which generated the new data sample, and E_θ be the expectation when the actual parameter is θ. It results from (13) that:

$$\mathrm{E}_\theta(\zeta_n(\theta_0)) = 0 \quad \text{iff } \theta = \theta_0 \tag{15}$$

In other words, vector $\zeta_n(\theta_0)$ in (14) has zero mean in the absence of change in θ, and nonzero mean in the presence of a change (damage). Consequently it plays the role of residual. Moreover, the residual can be shown to be asymptotically Gaussian.

For testing if $\theta = \theta_0$, a χ^2–test is employed to decide residual ζ_n is significantly different from zero or not, which should be compared to a threshold:

$$\chi_n^2 = \zeta_n^T \hat{\Sigma}^{-1} \hat{J} (\hat{J}^T \hat{\Sigma}^{-1} \hat{J})^{-1} \hat{J}^T \hat{\Sigma}^{-1} \zeta_n \tag{16}$$

where $\hat{J}$ in (16) is a consistent estimate of $J(\theta_0)$, the sensitivities of the residual with respect to the monitored parameters; $\hat{\Sigma}$ is a consistent estimate of $\Sigma(\theta_0)$ the asymptotical residual covariance.

3.2 Non-parametric Test

In some cases, it is difficult to make identification and obtain the modal parameters of the structure exactly from identification algorithm for some reasons. For example, because the earthquake will excite the structure at certain frequencies, not only the structural frequencies will be found among the identification results but also the frequencies of the earthquake wave. In this case, it maybe of interest to replace the above parametric approach with a non parametric one, based on an empirical null space computed on a reference data set instead of a reference modal signature. Namely with this approach, the above algorithm only applies with the time series, but not with the structural modal parameters. Such a null space may result from a SVD of the empirical Hankel matrix built on the covariances of the reference data set (indexed with 0):

$$\hat{S}_0^T \hat{H}_{p+1,q}^{(0)} = 0 \tag{17}$$

Such a non parametric detection approach based on (17) is used e.g. in ref.[9].

Based on new data from the (possibly damaged) system, the empirical Hankel matrix $\hat{H}_{p+1,q}$ is computed and the residual then writes:

$$\hat{\zeta}_n = \sqrt{n}\,\mathrm{vec}(S_0{}^T \hat{H}_{p+1,q}) \tag{18}$$

There is no sensitivity matrix to take care of, and the χ^2-test statistics boils down to:

$$\hat{\chi}_n^2 = \zeta_n^T \hat{\Sigma}^{-1} \zeta_n \tag{19}$$

3.3 Stochastic Bootstrap Method for Computation of Covariance

Because of the small size of structural responses under earthquake excitation, it is hard to obtain a correct estimate for the residual covarianc in the damage detection algorithm as in (16). Resampling methods are the more general way to overcome this difficulty. In this paper, we use a stochastic bootstrap approach to solve this problem. Bootstrap method is a resampling method for statistical inference [10]. It is commonly used to estimate confidence intervals but is can also be used to estimate bias and variance of an estimator or calibrate hypothesis tests. The key idea is to replace analytical calculations of biases, variances, confidence and prediction intervals, and other measures of uncertainty, with computer simulation from a suitable statistical model. In a nonparametric situation this model consists of the data themselves, and the simulation simply involves resampling the existing data.

Concretely, the structural response under earthquake excitation in this work is processed as follows:

1) Consider a set of data containing n samples: $x=(x_1, x_2, \ldots x_n)$, divide the data into B segments, thus each segment contains n/B samples: S_1: ($x_1, x_2, \ldots, x_{n/B}$), S_2: ($x_{n/B+1}, x_{n/B+2}, \ldots, x_{2n/B}$), …, S_B: ($x_{n(B-1)/B+1}, x_{n(B-1)/B+2}, \ldots, x_n$);

2) Permute all B segments, and allow repeat in the different positions, namely resampling, such as:

$$\underbrace{S_1, S_1, S_2, S_3, S_4, \ldots, S_B}_{B},\ \underbrace{S_1, S_2, S_2, S_3, S_3, \ldots, S_B}_{B}, \ldots$$

where the samples is in the same order within each segment; thus there are B^B possible combinations, called the ideal bootstrap samples;

3) Compute the different Σ_i with the “fake” signals, i.e. each bootstrapped signal will yield to one estimate of the residual covariance and we get the final estimate by averaging:

$$\hat{\Sigma} = \frac{1}{m}\text{sum}(\hat{\Sigma}_1 + \hat{\Sigma}_2 + \cdots \hat{\Sigma}_m) \tag{20}$$

In this bootstrap approach, the number of "fake" signals m is a parameter to tune.

4 APPLICATION

The proposed modal identification and detection methods of Sections 2 and 3 have proven useful in a number of application examples. However, the main source of dynamic excitation for structures in all these applications is ambient excitation. In order to test the validity of the proposed algorithm for processing the seismic signal, in this paper, the algorithm is applied to a real bridge, the Painter Street Overpass Bridge. The first reason for selecting the PSO Bridge for a case study was the availability of a rich data bank of previous earthquakes.

4.1 Description of Painter Street Overpass Bridge

The PSO Bridge is a two span, pre-stressed concrete box-girder bridge constructed in 1973 over the four-lane US Highway 101 in Rio Dell, Northern California. Its construction is typical of the type of California bridges used to span two or four lane highways, shown on Figure 1. The bridge is 15.85 m wide and 80.79 m long. The deck is a multi-cell box girder, 1.73 m thick and is supported on monolithic abutments at each end and two-pier bent that divides the bridge into two spans of unequal length; one of the spans is 44.51 m long and the other is 36.28 m long. The abutments and piers are supported by concrete friction piles and are skewed at an angle of 38.9 degrees. Longitudinal movement of the west abutment is allowed by means of a thermal expansion joint at the foundation level. The piers are about 7.32 m high, each supported by 20 concrete friction piles. The east and west abutments are supported by 14 and 16 piles, respectively.

The bridge was instrumented in 1977 as a collaborative effort between the California Strong Motion Instrumentation Program of the Division of Mines and Geology and CALTRANS to record and study strong motion records from selected bridges in California. Twenty strong motion accelerometers were installed on the bridges as shown in Figure 2.

Figure 1: Painter Street Overpass Bridge.

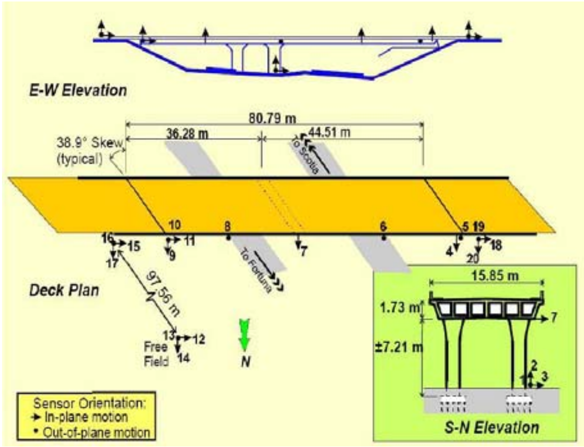


Figure 2: PSO Bridge sensor layout.

4.2 Description of the Recorded Earthquakes

Some earthquakes time series were recorded using the sensors instrumented on the bridge. In this paper, we select the 10 most significant earthquakes recorded for the analysis, which are summarized in Table 1. One of the typical structural responses under earthquake 92ML6.9 is shown in Figure 3. In this work, we still consider that at most a small damage occurred in the bridge since it still work normally after each earthquake. According to the theory, the proposed algorithm should be applied to the linear portion of the signal. Usually, covariance subspace handle non stationary in the signal just fine, but the available low number of sample prevents such smoothing and some care is needed. Normally, according to the procedure in earthquake, the time series contains four parts approximately as shown in Figure 3. In Part I, the beginning part, the structure is undamaged and linear, but the structural response is non-stationary because of the earthquake excitation. In Part II, the damaged part, the damage potentially increases, and the structural response is very non-stationary. In this work, we still consider that at most a small damage occurred in the bridge since it still work normally after each earthquake and no large damage occurred. This part of signal is very short. In Part III, the earthquake excitation has stopped, so the structure will recover its linear behavior gradually. In Part IV, the free-vibration part, the structure has recovered its linearity, and keeps on vibrating with much lower excitation. Part III and IV are used as test signals.

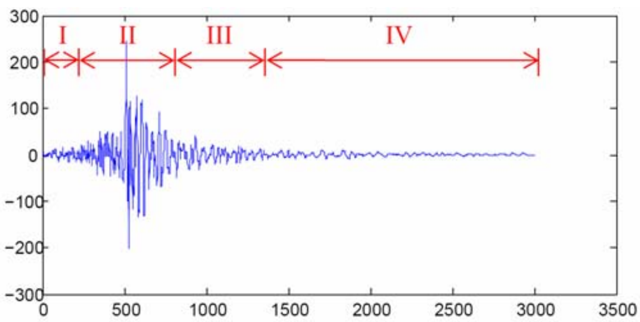


Figure 3: Typical structure response under earthquake excitation.

Event Code	Earthquake	Date	Mag. (M_L)	Epic. Dist. (km)	FF Accel. (g)	Struct. Accel. (g)	Sample Length
80ML6.9	Trinidad Offshore	8 Nov 1980	6.9	88	0.15	0.17	1100
82ML4.4	Rio Dell	16 Dec 1982	4.4	15	/	0.42	1082
83ML5.5	Eureka	24 Aug 1983	5.5	61	/	0.22	1037
86_1ML5.1	Cape Mendocino – 1	21 Nov 1986	5.1	32	0.43	0.40	1096
86_2ML5.1	Cape Mendocino – 2	21 Nov 1986	5.1	26	0.14	0.35	1100
87ML5.5	Cape Mendocino	31 Jul 1987	5.5	28	0.14	0.34	1100
92ML6.9	Cape Mendocino – Petrolia	25 Apr 1992	6.9	6.4	0.54	1.09	3000
92ML6.2	Cape Mendocino – Petrolia (AS1)	26 Apr 1992	6.2	6.2	0.52	0.76	3001
92ML6.5	Cape Mendocino – Petrolia (AS2)	26 Apr 1992	6.5	6.4	0.26	0.31	1500

Table 1: Significant earthquakes recorded at PSO bridge (1977-1992) [11].

4.3 Identification results

From the singular values of the spectral densities the five modes indicated in Figure 4 have been identified.

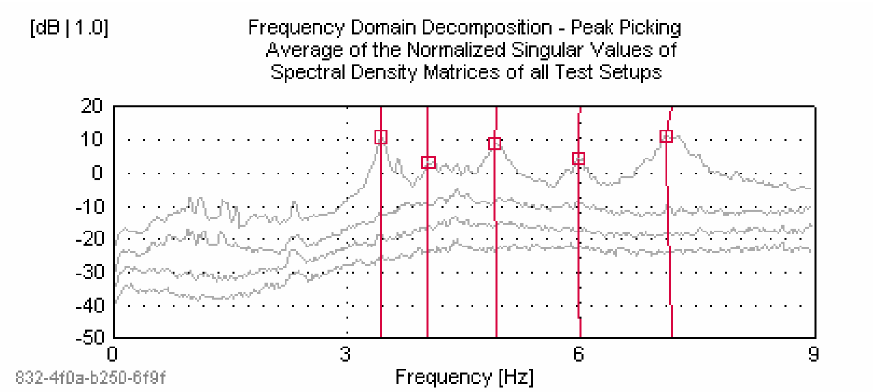


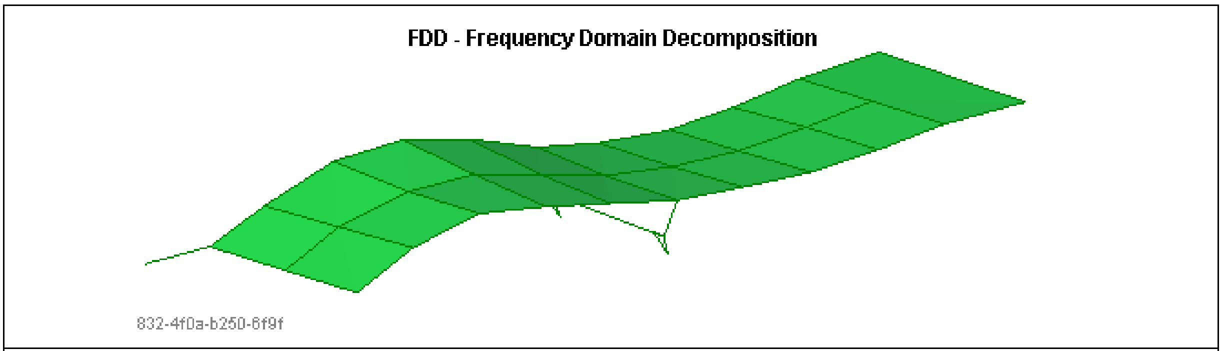
Figure 4: The Frequency Domain Decomposition (FDD) technique. The diagram is displaying the first 4 singular value lines as well as the peak points where the five first structural modes have been picked.

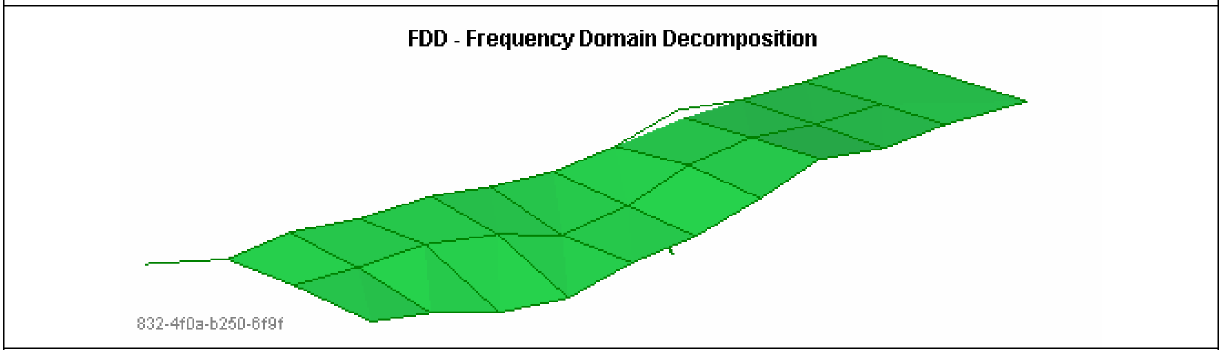
The natural frequencies of the five identified modes are:

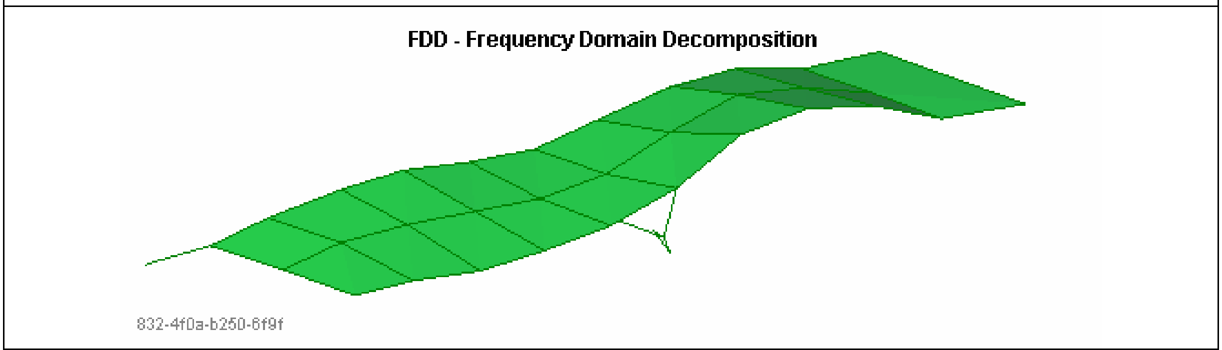
Mode Number	Natural Frequency (Hz)
1	3.44
2	4.04
3	4.19
4	5.99
5	7.17

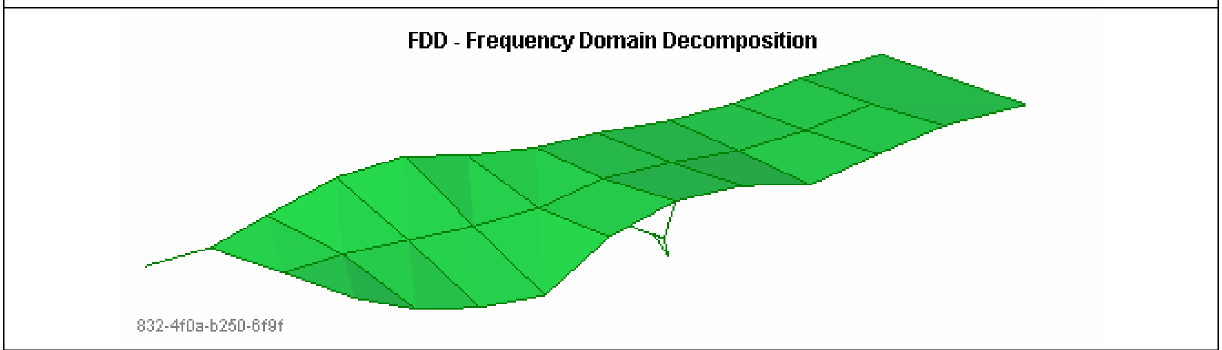
Table 2: Estimated natural frequencies of the first 5 structural modes.

The corresponding mode shapes of the five modes are shown below.









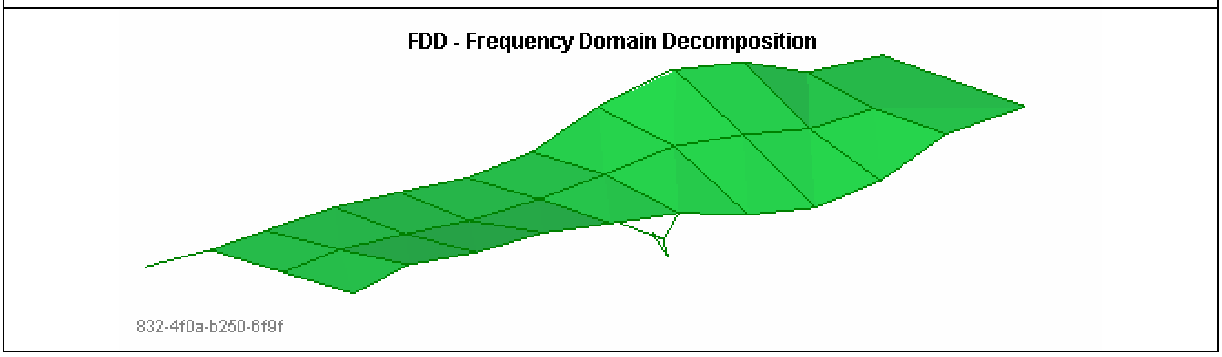
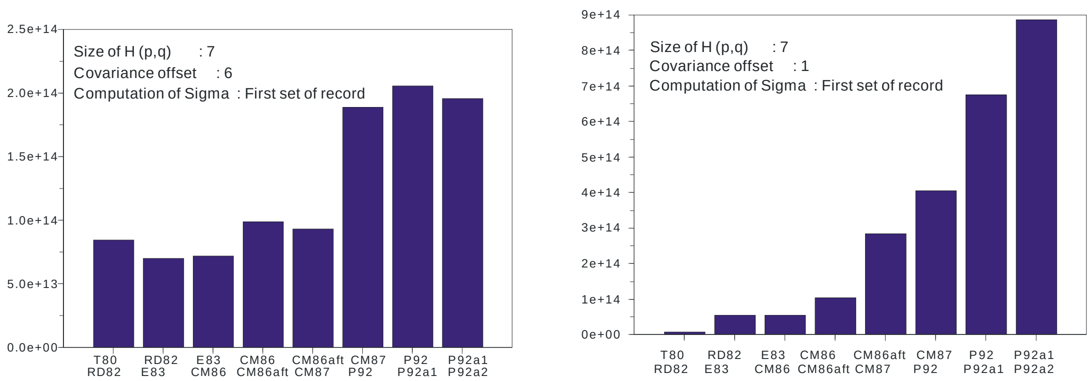


Figure 5. The mode shapes of the five structural modes.

4.4 Detection results

As amentioned above, the damage occurred to the structure is small, moreover we consider that the structure is increasingly damaged from its original state after each earthquake scenario. For the very short samples, as shown in Table 1, two successive earthquake records are joined together for computation of the Hankel matrices to increase the available number of samples. As shown below, this does not imply any delay in the detection. The first two earthquakes 80ML6.9 plus 82ML4.4 are considered the reference.

Finally, as stated above, the proposed damage detection method was applied to the reference data firstly, namely computing the estimate of $\hat{\Sigma}$ in (20). Then the χ^2-test is evaluated for each damage scenario. The result is shown in Figure 6, where the x and y axis represent respectively the earthquake records two by two, and the χ^2-test value, that is the seismic damage index. Notice that the regular increase in the damage detection value could yield to some measure of the damage level. The left figure shows that the proposed method does not work without stochastic bootstrap approach. In the right figure, the damage index shows that the damage is more severe with each new earthquake. The result of the experiment indicates that the proposed method is able to assess the seismic damage in the example bridge.



(a) without bootstrap method

(b) with bootstrap method

Figure 6: Results of seismic damage detection.

5 CONCLUSIONS

This paper presents a seismic damage detection method based on a stochastic subspace- from output-only data. A χ^2-test is used to obtain a damage decision. Especially for processing of the earthquake data, non-parametric test and stochastic bootstrap algorithms are used, the former built on empirical null space and the latter is used to compute the χ^2-metric . This seismic damage detection method was tested on a real bridge. The results indicate that the algorithm is able to detect the seismic damage on such earthquake records.

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