

Challenges in the Application of Stochastic Modal Identification Methods to a Cable-Stayed Bridge

Filipe Magalhães¹; Elsa Caetano²; and Álvaro Cunha³

Abstract: This paper presents an analysis of the data collected in the ambient vibration test of the International Gadiana cable-stayed Bridge, which links Portugal and Spain, based on different output-only identification techniques: peak-picking, frequency domain decomposition, covariance-driven stochastic subspace identification, and data-driven stochastic subspace identification. The purpose of the analysis is to compare the performance of the four techniques and evaluate their efficiency in dealing with specific challenges involved in the modal identification of the tested cable-stayed bridge, namely the existence of closely spaced modes, the perturbation produced by the local vibration of stay-cables, and the variation of modal damping coefficients with wind velocity. The identified natural frequencies and mode shapes are compared with the corresponding modal parameters provided by a previously developed numerical model. Additionally, the variability of some modal damping coefficients is related with the variation of the wind characteristics and associated with a component of aerodynamic damping.

DOI: 10.1061/(ASCE)1084-0702(2007)12:6(746)

CE Database subject headings: Stochastic models; Vibration; Bridges, cable-stayed; Portugal; Spain.

Introduction

The design and construction of progressively larger and more complex structures and the continuous aging and subsequent structural deterioration of existing structures made structural engineers feel it necessary to develop appropriate experimental tools for the accurate identification of the most relevant structural properties (static and dynamic) in order to provide reliable data to support the calibration, updating, and validation of finite element models developed during the design phase or in the context of rehabilitation works and to evaluate the health condition of an existing structure.

The first approach to identifying dynamic structural properties taken by civil engineering researchers was to use important previous developments made in system identification and experimental modal analysis in electrical and mechanical engineering in order to accurately identify the main dynamic properties of civil structures. However, it was difficult to apply controlled excitations to large civil structures, and so a new dynamic testing approach, exclusively based on the measurement of the structural response to ambient excitations and the application of suitable stochastic modal identification methods, was developed. The success of this so-called ambient vibration test technique was made

possible only by the remarkable technological progresses in the area of transducers and analog-to-digital converters, which allowed the measurement of very low accelerations, and by the development of algorithms that could accurately identify the modal parameters using only the measured responses.

To enhance the quality of the information extracted from the data collected in ambient vibration tests, many research works have been developed, and so several powerful identification methods are presently implemented in user-friendly software (Branden et al. 1999; LMS Int. 2005; SVS 1999–2004) that can be applied in the dynamic identification of structures to provide good results. However, the performance of the existing identification methods has been compared in only a limited number of studies, and even fewer of these studies use data from complex large civil structures, such as cable-stayed bridges.

In this context, this paper presents a comparative analysis of the application of some of the most recent and powerful output-only identification techniques to a cable-stayed bridge. The techniques compared are peak-picking (PP), frequency domain decomposition (FDD), covariance-driven stochastic subspace identification (SSI-COV), and data-driven stochastic subspace identification (SSI-DATA). The study's main objective is to analyze the performance of the applied techniques and evaluate their efficiency in dealing with specific challenges involved in the modal identification of the International Gadiana cable-stayed bridge. In particular this study addresses the existence of closely spaced modes, the perturbation produced by the local vibration of stay-cables, and the variation of modal damping ratios with wind velocity.

Description of the Bridge and Numerical Modeling

The International Gadiana Bridge (Fig. 1) is a concrete cable-stayed bridge, located in the southeast of Portugal and connecting Portugal and Spain over the Gadiana River. The bridge, designed by Portuguese engineer Câncio Martins, was open to traffic in

¹Assistant, Faculty of Engineering of Univ. of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal. E-mail: filipema@fe.up.pt

²Associate Professor, Faculty of Engineering of Univ. of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal. E-mail: ecaetano@fe.up.pt

³Full Professor, Faculty of Engineering of Univ. of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal. E-mail: acunha@fe.up.pt

Note. Discussion open until April 1, 2008. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on February 6, 2006; approved on November 15, 2006. This paper is part of the *Journal of Bridge Engineering*, Vol. 12, No. 6, November 1, 2007. ©ASCE, ISSN 1084-0702/2007/6-746-754/\$25.00.

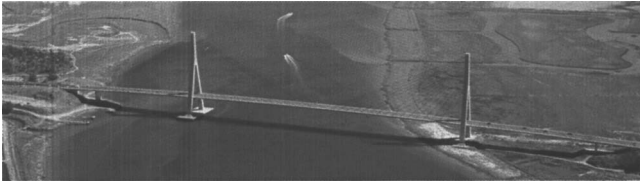


Fig. 1. Aerial photo of bridge

1991 and was one of the first cable-stayed bridges with a moderate span constructed in Portugal. The bridge is composed of a central span of 324 m, two lateral spans of 135 m, and two transition spans of 36 m, resulting in a total length of 666 m (Fig. 2). The deck is a prestressed concrete box section, 18 m wide and 2.5 m high, with internal concrete bracings at 4.5 m intervals. It is supported at the towers and suspended by stay-cables. These are arranged in a semi-fan, in a total of 32 pairs anchored at the top of each tower and at the deck at 1.8 m and 9 m intervals, respectively. The two A-shaped concrete towers, which provide vertical and lateral supports for the deck, are 100 m high and are supported by pile groups, with 2.0 m diameter piles, founded on schist rock. The stay-cables are bundles of 15 mm monostrands, comprised of between 22 and 55 strands. Each bundle has clamped collars at mid-length for the shortest cables (chord length of less than 70 m) and at the thirds of the length for the other cables. No external pipes were used to encase the strands, making them more vulnerable to wind excitation.

Because its stay-cables were frequently observed to oscillate at high amplitude under winds of moderate speed, this bridge was selected as the prototype of a research project, funded by the Portuguese Foundation for Science and Technology, which aims to develop and install a continuous monitoring system (Caetano et al. 2005) to study, in particular, the vulnerability of the cables to parametric excitation (Caetano and Cunha 2003).

The numerical analysis of the bridge's structural behavior was based on a finite element model supplied by the bridge designer, in which the deck and towers are idealized as 3D beam elements and the stay-cables as single truss elements, connected to the deck and to the towers through a series of rigid links, in order to correctly simulate the real geometry of the bridge. This model was then improved so as to incorporate the geometric nonlinear behavior under permanent loads, to accommodate the real cable tensions, measured on site (Caetano and Cunha 2003), and to take into account the hardening of the concrete caused by aging. The results provided by the numerical model are presented together with the modal parameters estimated by the peak-picking method (Figs. 5–7). These numerical results, namely the natural frequencies and mode shapes, were essential to define certain key aspects of the ambient vibration test, such as the time of acquisition, the sampling frequency, and the position of the reference section.

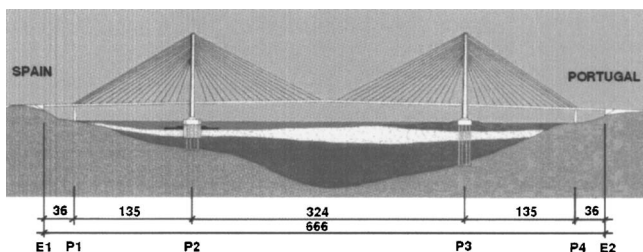


Fig. 2. Side view of bridge (distance in meters)

Ambient Vibration Test Description

An ambient vibration test of a cable-stayed bridge uses testing equipment satisfying specific requirements. In this case, the measurement of the bridge's response to ambient excitation necessitated that sensitive accelerometers be used and the low values of the natural frequencies required the use of accelerometers with linear responses at low frequencies, close to DC. Finally, the considerable extension of the bridge would make the use of electric cables rather cumbersome and time consuming. Therefore, 4 triaxial 18-bit strong motion recorders were used in the present application. These devices are comprised of sensitive internal force balance accelerometers (linear behavior from DC to 100 Hz), 18 bit analog-to-digital converters (to guarantee high sensitivity), batteries that enable autonomy for one day of tests, memories materialized by removable compact flash cards that permit a fast download of the acquired data, and external GPS sensors to ensure an accurate time. Thus, they can work independently and synchronously, avoid the use of cables, and minimize the labor associated with the preparation of the dynamic test.

In the ambient vibration test of the International Guediana Bridge, two recorders, permanently located at section 17 (see Fig. 3) on both sides of the deck (locations 17U upstream and 17D downstream), served as references. The other two recorders scanned the bridge deck and towers in 28 consecutive setups, measuring the acceleration along the 3 orthogonal directions for all the points represented in Fig. 3. For each setup, time series of 21 min (1260 s) were collected, corresponding to approximately 500 cycles of the longest vibration period of the structure. The sampling frequency was 100 Hz, a value that is imposed by the filters of the acquisition equipment and that is much higher than necessary in the test of a cable-stayed bridge, where the most relevant natural frequencies are below 3 Hz.

In parallel with the measurement of accelerations, the wind speed was monitored at deck level. Discrete measurements were performed with a propeller anemometer. A mean wind velocity was associated with each setup, corresponding to a measuring duration of 21 min. It was observed that the wind velocity presented small variations during each of the 3 consecutive days of the test but a significant variation for the different days. A mean wind velocity of 2 m/s was observed on the first day (expressed by the mean of the wind velocities collected for the various setups). On the second day, a mean velocity of 9 m/s was recorded, and in the morning of the last day of tests, a mean wind velocity of 14 m/s was reached. The amplitudes of the transversal and longitudinal accelerations are dependent on the wind velocity. Consequently, the root mean square (*rms*) values of the longitudinal acceleration varied from 0.24 mg (day 1) to 0.46 mg (day 3) and the *rms* values of the transversal acceleration varied from 0.35 mg (day 1) to 0.74 mg (day 3). The vertical acceleration is also dependent on the wind velocity. However, it depends mostly on the intensity of heavy traffic, which was moderate on the first test day (Saturday), especially low on the second day (Sunday), and high on the third day (Monday). This is reflected by the *rms* values of the observed vertical acceleration: 1.54 mg on Saturday, 1.27 mg on Sunday, and 3.41 mg on Monday.

Stochastic Modal Identification

Introduction

The stochastic modal identification methods, also designated as output-only modal identification methods, identify structural

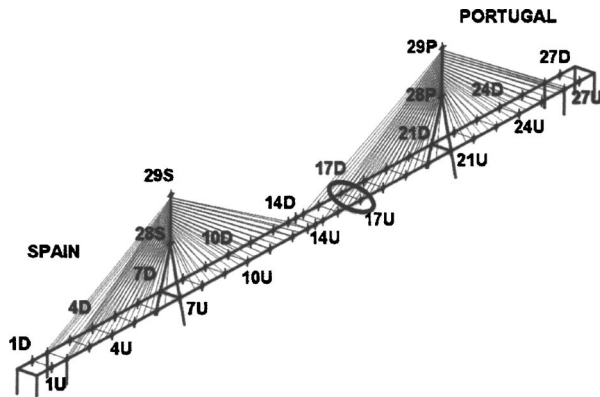


Fig. 3. Measurement points used in ambient vibration test (ellipse indicates reference locations)

modal properties using only responses (output) and assuming some appropriate hypotheses in terms of the excitation idealization. Since ambient excitation commonly has a multiple input nature and wide band frequency content, stimulating a significant number of modes of vibration, the stochastic identification methods assume, for simplicity, the excitation input as a zero mean Gaussian white noise. This means that the real excitation can be interpreted as the output of a suitable filter excited by that white noise input. Modeling the behavior of the filter-structure system, one may conclude that some additional computational poles, without structural physical meaning, appear as consequences of the white noise assumption.

The classification of the large variety of stochastic modal identification methods presently available is usually based on the type of data used in its application, which can be the acceleration time series, the correlations, or the spectra. The methods that use the spectra are called frequency domain methods, those that use correlations and time series are designated as time domain methods.

In this work two methods in the frequency domain are described and applied: peak-picking and frequency domain decomposition. These are the methods most commonly used in civil engineering applications. Recently, a new alternative method in the frequency domain has been developed, also leading to good results. This method is called PolyMax and is based in the fitting of theoretical spectra to experimental spectra of the structure responses (Peeters and Auweraer 2005).

Concerning the methods in the time domain, this work focuses on those that perform the identification of state space models. From that group, two methods will be briefly described and applied: the covariance-driven stochastic subspace identification and the data-driven stochastic subspace identification. It is worth noting that other methods in the time domain use different models, such as the ARMAV (autoregressive moving average vector) and ARV (autoregressive vector) models (Andersen 1997; Piombo et al. 1993), but these methods have never reached an acceptable level of robustness and efficiency for civil engineering applications.

Peak-Picking Method

The PP or BFD (basic frequency method) method, was the first method used to perform the modal identification of civil engineering structures based on ambient vibration responses. Former applications of the method, using a procedure different from the

current one, are described in Crawford and Ward (1964), McLamore et al. (1971), Trifunac (1972), Abdel-Ghaffar and Housner (1978).

The theoretical background of the PP method was developed by Bendat and Piersol (Bendat and Piersol 1971). Felber (Felber and Cantieni 1996; Felber 1993) developed an application procedure implemented automatically and extensively used. The method was successfully applied in the analysis of several structures, including another Portuguese cable-stayed bridge, the Vasco da Gama Bridge (Cunha et al. 2001).

The validity of the PP method implies the existence of low modal interaction. A common procedure to improve the quality of the estimates involves separating different types of modes by combining signals. For bridge structures, the half difference and half sum of the vertical acceleration components measured at two opposite extremes of a particular deck cross section emphasize the contribution of torsional and vertical bending signal components, respectively. So, previous to the application of the PP method, three combined signals were calculated for each instrumented bridge section: the half sum of vertical acceleration components, the half difference of vertical acceleration components, and the half sum of transversal acceleration components.

A basic step of the method is the evaluation of spectral estimates of the ambient structural responses from the measured time series. This evaluation can be completed using the Welch procedure (Welch 1967), which involves (1) dividing the response records into several segments (eventually with some overlap); (2) applying a signal processing window to the data segments to reduce leakage; (3) computing the discrete Fourier transform (DFT) of the windowed segments; (4) computing the auto and cross spectra using the DFT of the windowed segments; and finally, (5) averaging the spectra associated with each time segment.

Considering the application to the Guadiana Bridge, the previously combined time records were first low-pass filtered and down-sampled at a rate of 10 Hz. Then, the spectra were estimated using time segments with 2,048 points (204.8 s). Based on an overlap of 50%, spectral averages over 10 time segments, with a frequency resolution of 0.0048 Hz, were obtained.

Beyond that, all the autospectra were normalized and averaged to obtain an average normalized power spectral density function (ANPSD), with peaks that show the structure's resonant frequencies. Assuming the excitation does not contain any artificial periodic contribution, the damping is small and the natural frequencies are well separated, the abscissas of these peaks are the natural frequencies of the dynamic system.

Figs. 4(a-c) present the average normalized power spectral density functions of the three combined signals. Inspection of these results indicates that some peaks of these spectra are not related to global mode shapes but to local cable modes of vibration stimulated by the wind. So, to distinguish global natural frequencies from local cable frequencies, autospectral density functions of the ambient response at the reference section, associated with different wind velocities, were calculated. (Fig. 4(d) represents the power spectral density functions of the half sum of vertical accelerations for three different wind velocities: 2, 10, and 14 m/s).

Using these spectra, a significant number of natural frequencies could be identified (marked in the ANPSD, at Fig. 4). One of the natural frequencies (0.845 Hz) was identified from the inspection of power spectral density functions of the ambient response at several sections, measured in setups associated with low wind

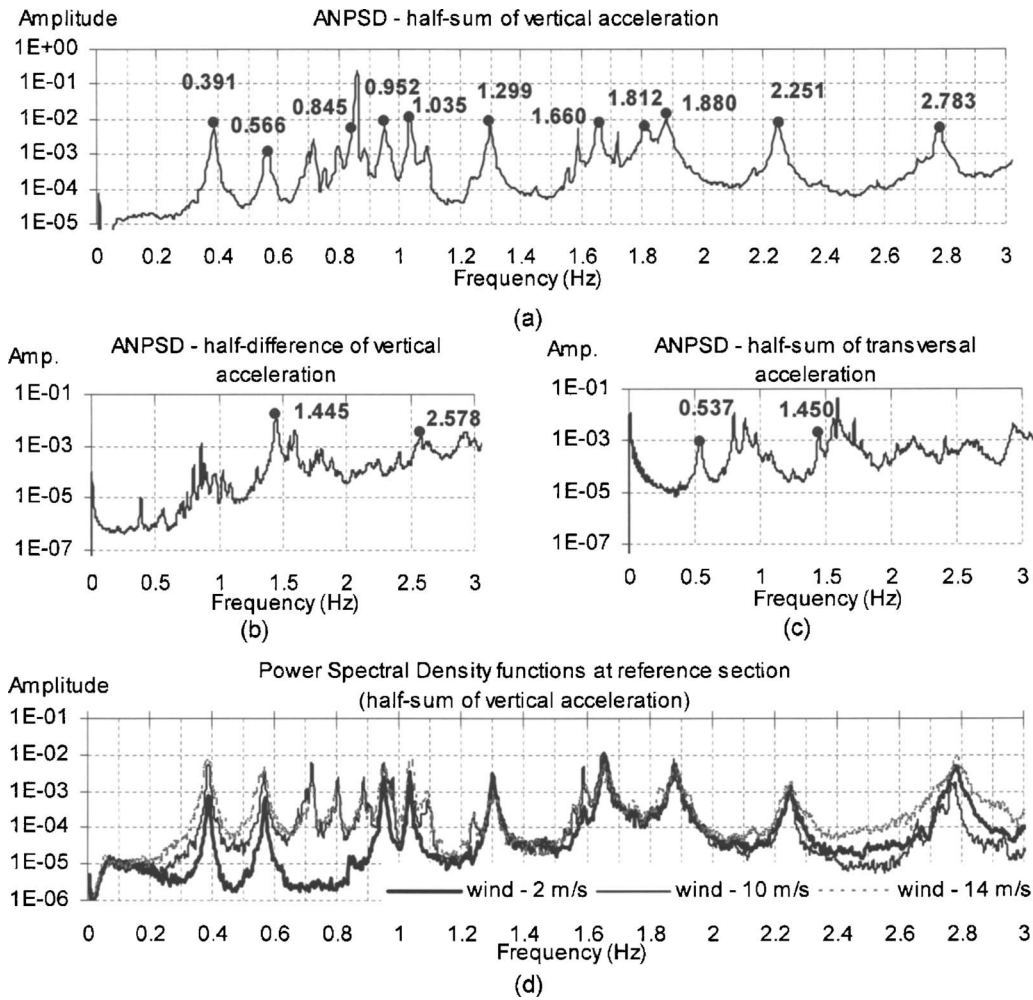


Fig. 4. Average normalized power spectral density functions (a,b,c) and power spectral density functions at reference section of half-sum of the vertical acceleration for different wind velocities (d)

velocities, other than the reference section, given that the corresponding mode shape has a nodal point in the vicinity of that location.

For the identification of the mode shapes, it can be shown that the ratio between a cross spectra ($S_{i,j}$) and an autospectra ($S_{j,j}$) (ambient response transfer function), evaluated for a resonant frequency, is a complex value with an amplitude that provides an estimate of the ratio between the modal coordinates at points i and j of the mode shape associated with the considered resonant frequency. The phase of the complex number indicates whether the points are moving in the same sense (zero phase values) or in opposite senses (π rad phase values) (Felber 1993).

Therefore, choosing a measurement point as reference (ref), the mode shapes can be identified looking at the amplitude and phase of the ratio between the cross spectra ($S_{i,ref}$) and the autospectra ($S_{i,i}$) evaluated for each natural frequency. As the modal amplitude of the reference point appears in the denominator, it shouldn't be zero. So, it is important to avoid the location of a reference point close to a node of a mode that it is important to identify. In the present analysis, the reference section was selected based on the mode shapes provided by the previously developed numerical model. If it is difficult to predict the configuration of the mode shapes, a second reference should be used.

Figs. 5–7 present the configurations of the most relevant identified mode shapes of the deck, together with the corresponding

numerical modes. Vertical and torsion mode shapes are represented using a lateral view of the deck, and transversal mode shapes are represented using a top view. The symbol • defines the position of towers and piers. The comparison of the experimental and numerical modes evidences a good correlation, both in term of natural frequencies and mode shape configurations.

The modal configuration associated with the frequency 0.845 Hz is not as smooth as the others because it has low modal amplitude at the reference section. A similar observation can be made for the transversal modes; in fact, that is a consequence of the lower signal/noise ratio in that direction. In effect, the amplitude of the measured transversal acceleration was around 1/5 of the vertical component.

Two pairs of modes have very close frequencies (second vertical mode/first transversal mode and first torsional mode/second transversal mode), which were successfully identified by this method owing to the independent analysis of the precombined time series. It is worth noting that the numerical model was important for identifying the second lateral mode as an independent mode and not a lateral deflection shape associated with the 1st torsion mode.

In the context of the PP method, the identification of modal damping coefficients is usually performed using the half-power bandwidth method (Bendat and Piersol 1980), but the resulting

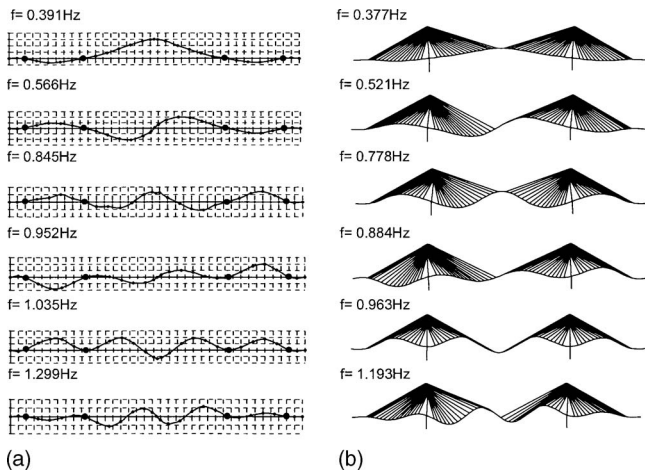


Fig. 5. Identified (a) versus calculated; (b) vertical modes

estimates are not reliable. So, in this work modal damping coefficients are estimated using only more advanced stochastic identification techniques.

Frequency Domain Decomposition

The basic principles of the frequency domain decomposition (FDD) method were introduced by Prevosto (1982); Corrêa and Costa (1992). The method was, however, better detailed and systematized by Brincker et al. (2000), with the objective of overcoming the PP method limitation related to the separation of closely spaced modes, and it was subsequently enhanced to extract modal damping ratio estimates (Brincker et al. 2001).

The first step of this method is the construction of a spectrum matrix for each test setup, with a number of lines equal to the number of measurement points in each setup and with as many columns as the number of points elected as references. Each column contains the cross spectra between the structural response at all the measured points and the corresponding response at a reference point.

In the present application four time segments are considered for each instrumented section: upstream and downstream vertical accelerations, lateral acceleration, and longitudinal acceleration. So, for each setup, spectrum matrices with 8 rows and 4 columns were calculated using the same procedure and parameters as used for the peak-picking.

It can be shown that under some assumptions (white noise excitation, low damping, and orthogonal mode shapes for close modes) the singular values of the spectrum matrix, in the vicinity of each resonant frequency, are autospectral density functions of a single degree of freedom system with the same frequency and damping as the different structure vibration modes (Brincker et al. 2000). Therefore, the spectral matrix (S_y) is decomposed, at each frequency (ω_i), in singular values and vectors using the singular

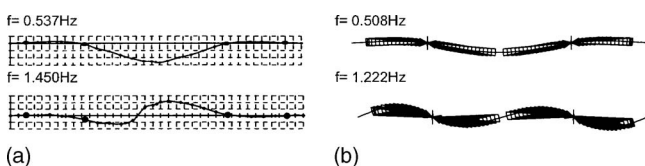


Fig. 6. Identified (a) versus calculated; (b) transversal modes

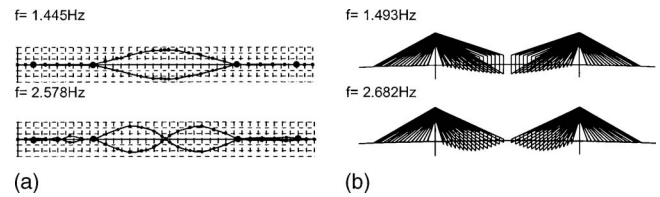


Fig. 7. Identified (a) versus calculated; (b) torsional modes

values decomposition (SVD) algorithm. This operation is described by Eq. (1), where the matrix U_i contains the singular vectors and S_i is a diagonal matrix that contains the singular values [$(\bullet)^H$ refers to the transpose conjugate].

$$S_y(\omega_i) = U_i \cdot S_i \cdot U_i^H \quad (1)$$

The analysis of singular value functions calculated from the 28 spectrum matrices associated with the 28 performed setups allowed the identification of 12 resonant frequencies. These are present in all setups in the frequency range of analysis, 0–3 Hz, corresponding to deck vibration modes. Some additional resonant frequencies appear after the 13th setup, when the wind velocity increased. These are related with the cable vibrations stimulated by the wind.

The application of the FDD method identified two modes with close frequencies (0.548 and 0.568 Hz) but not the modes with almost coincident frequencies, which were identified based on the application of the peak-picking method (1.445 Hz and 1.450 Hz). The frequency of 0.845 Hz was not identified because of the proximity of the reference section to one node of the mode shape.

The autospectra density functions formed by the singular values of the spectrum matrices of the 28 setups were then used to calculate autocorrelations functions, associated with the different modes of the structure, applying an inverse FFT. From these functions, it is straightforward to identify the modal damping coefficients and obtain enhanced estimates of the natural frequencies. These frequencies were calculated from the time intervals between each zero crossing. The modal damping coefficients were estimated by adjusting an exponential decay to the relative maxima of the autocorrelation functions.

Columns 5 and 6 of Table 1 present, for each mode, the mean of the 28 identified frequencies (f) and modal damping ratios (ξ). The standard deviations of the identified natural frequencies oscillate between 0.002 Hz (in the 7th mode) and 0.061 Hz (in the 2nd mode). The standard deviations associated with the modal damping ratios vary between 0.11 % (7th mode) and 0.79% (2nd mode) because there is usual uncertainty associated with the estimation of damping and because this parameter varies with the wind velocity, motivated by aerodynamic damping. As referred above, the test was performed in 3 days and during this period the mean wind velocity increased from 2 m/s to 14 m/s.

The mode shapes are estimated from the singular vectors of the spectrum matrices evaluated at the identified resonance frequencies and associated with the singular values that contain the peaks. In each setup 8 modal components are calculated and are grouped together using the reference points.

Fig. 8 shows the configurations of the first and second vertical bending modes and two views of the first torsion mode, plotted with the Artemis software (SVS 1999–2004). To obtain these configurations, the measurements performed at two levels along the towers were also considered. Careful inspection of the configuration of the first torsion mode shows that the torsion movement is coupled with a transversal bending component of the deck, which

Table 1. Comparison of Results Obtained with Applied Techniques

Mode	Mode type	Numerical	PP	FDD		SSI-COV		SSI-DATA		Autocorrelation ^a	
		f [Hz]	f [Hz]	f [Hz]	ξ [%]	f [Hz]	ξ [%]	f [Hz]	ξ [%]	f [Hz]	ξ [%]
1	Vertical	0.377	0.391	0.388	1.37	0.388	1.09	0.387	1.09	0.390	1.30
2	Transversal	0.508	0.537	0.548	2.23	0.538	2.06	0.537	2.28	0.539	2.11
3	Vertical	0.521	0.566	0.568	1.2	0.565	1.18	0.565	1.21	—	—
4	Vertical	0.778	0.845	—	—	—	—	—	—	—	—
5	Vertical	0.884	0.952	0.950	0.69	0.951	0.53	0.954	0.47	—	—
6	Vertical	0.963	1.035	1.033	0.51	1.033	0.42	1.033	0.43	—	—
7	Vertical	1.193	1.299	1.299	0.54	1.299	0.47	1.298	0.47	—	—
8	Torsion	1.493	1.445	1.448	0.47	1.449	0.42	1.447	0.42	1.445	0.45
9	Transversal	1.222	1.450	—	—	—	—	—	—	—	—
10	Vertical	1.506	1.660	1.655	0.63	—	—	—	—	—	—
11	Vertical	1.684	1.812	—	—	—	—	—	—	—	—
12	Vertical	1.786	1.880	1.880	0.65	—	—	—	—	—	—
13	Vertical	2.140	2.251	2.248	0.59	—	—	—	—	—	—
14	Torsion	2.682	2.578	2.595	1.48	—	—	—	—	—	—
15	Vertical	2.644	2.783	2.779	0.46	—	—	—	—	—	—

^aMean values obtained from the autocorrelation functions used to estimate the variation of damping with the wind velocity.

has a similar configuration to that obtained for the second transversal mode by the peak-picking method and by the numerical model. This proves that this method did not individualize the two modes with almost coincident natural frequencies. The other modes are not represented because they are coherent with the ones identified by the peak-picking method.

Covariance-Driven Stochastic Subspace Identification

The stochastic subspace identification (SSI) methods are an interesting alternative to the previous frequency domain approaches. These methods rely on a stochastic state-space model that, in its discrete form and idealizing the excitation as a white noise, is represented by the following equations:

$$\begin{aligned} x_{k+1} &= A \cdot x_k + w_k \\ y_k &= C \cdot x_k + v_k \end{aligned} \quad (2)$$

where y_k =a column vector, with l components (number of measured outputs), that characterizes the output of the system at the time instant k , x_k =the state vector, which has n components (dimension of the state space model); w_k represents the noise used to simulate the ambient excitation and the model inaccuracies; and v_k =the noise that simulates the error introduced by the measurement system and also the ambient excitation. The matrix A ($n \times n$)=state transition matrix and completely characterizes the dynamics of the structure, while matrix C ($l \times n$)=output matrix and specifies how the internal states are transformed in outputs.

The covariance-driven SSI method (SSI-COV) uses the correlations of the structure responses to estimate the A and C matrices.

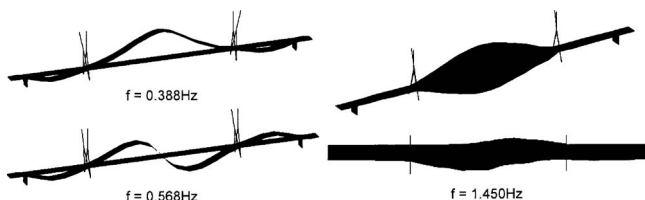


Fig. 8. Three identified mode shapes

ces. Three different procedures can be used to obtain these correlations; they can be obtained directly from the time series using Eq. (3), from previously calculated spectra, or using the random decrement (Asmussen 1997). The first alternative requires more computations but provides estimates without bias errors and is easy to implement. Consequently, it was the option chosen for this application

$$\begin{aligned} \hat{R}_{x_1, x_2}(j) &= \frac{1}{N} \cdot \sum_{k=1}^N x_1(k \cdot \Delta t) \cdot x_2(k \cdot \Delta t + j \cdot \Delta t) \\ &\text{with } j = 0, 1, 2, \dots, 2i - 1 \end{aligned} \quad (3)$$

The identification of matrices A and C is based on the factorization property of the correlations and relies on two important mathematical operations: singular value decomposition and the Moore-Penrose pseudoinverse (Peeters 2000). In the identification, a correlation matrix with the cross correlations of all the measured points can be used, or alternatively, to minimize the calculation effort, a correlation matrix containing just the cross-correlations with a selected subset of points can be employed. The former procedure was used in the present application.

After the state-space model is identified, the modal parameters are extracted from matrices A and C . In effect, the eigenvalue decomposition of A gives

$$A = \Psi \cdot \Lambda_d \cdot \Psi^{-1} \quad (4)$$

where Ψ ($n \times n$)=the eigenvector matrix; and Λ_d ($n \times n$)=a diagonal matrix containing the eigenvalues, μ_i , of the discrete state space model.

The eigenfrequencies, ω_i , and the damping ratios, ξ_i , are found from

$$\mu_i = e^{\lambda_i \Delta t} \quad \lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2} \omega_i \quad (5)$$

where λ_i =the eigenvalues of the continuous state-space model; and Δt =the sampling time interval. The mode shapes V ($l \times n$) are calculated with the following relation:

$$V = C \cdot \Psi \quad (6)$$

It is worth noting that the identification of the state-space model requires the definition of the order of the model. However, for

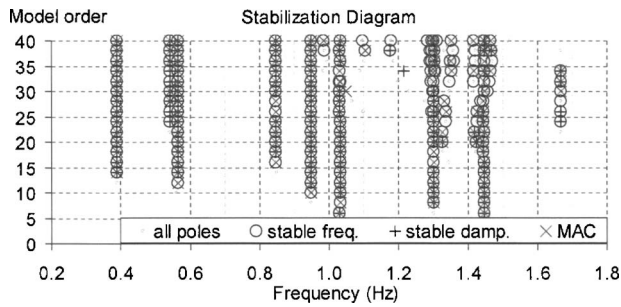


Fig. 9. Stabilization diagram of first setup for $i=50$ and considering these criteria to define stable poles: differences inferior to 1% for frequencies, differences inferior to 5% for damping ratios, and MAC coefficient higher than 0.99

real structures it is not possible to predict the order of the model that better fits the experimental data and more realistically characterizes the dynamic behavior of the structure. The most appropriate way to overcome this difficulty is to estimate the modal parameters using models with an order defined within an interval previously defined in a conservative way. The identified modal parameters are then represented in a stabilization diagram (Fig. 9). Such a diagram permits the parameters that are stable for models of increasing order—those that have structural meaning—to be distinguished. The others are associated only with the numerical modes that are important to model the noise always present in measured data.

In the present work, the method was applied, in a first instance, using the same data that was used with the frequency domain method (sampling frequency of 10 Hz). But it was concluded that the achievement of reasonable estimates for all modes would involve the use of models of very high order because several poles are necessary to model the local effects associated with vibrating cables. In models of high order the number of numerical poles is also high, thus it is more difficult to select the poles associated with global vibration modes of the structure. So, a second analysis was performed, in a narrower frequency range (0 to 1.67 Hz), to provide much better results with lower calculation effort.

Consequently, before the calculation of the correlation functions, the measured time segments were low-pass filtered and down-sampled at a rate of 3.33 Hz. In the calculation of the correlation functions the maximum time lag [$2i-1$ in Eq. (3)] has to be defined. This parameter has influence on two aspects: the maximum order of the estimated models equal to $i \cdot r$ (r being the number of points measured in each setup); and the quality of the stabilization diagrams and consequently of the estimated modal parameters. The increase of i also has the drawback of increasing substantially the calculation effort, so it is important to find a balance between the desired quality and the needed processing time. Taking that into account, $i=50$ was adopted in the present application, meaning that the segments of the correlation functions selected for the identification contain $2 \times 50 / 3.33 \times 0.391 \approx 12$ cycles of the lowest frequency. This choice for i , enables the estimation of models of order up to 200 (8 measurements in each setup). It is important to refer two aspects: if the down-sampling is not applied, it is necessary to adopt a higher i to have the same number of cycles, consequently, increasing the calculation effort dramatically, and the lowest value tested for i ($i=10$) allows the estimation of models of order up to 80, which is a reasonable number because only 10 structural modes are predicted by the

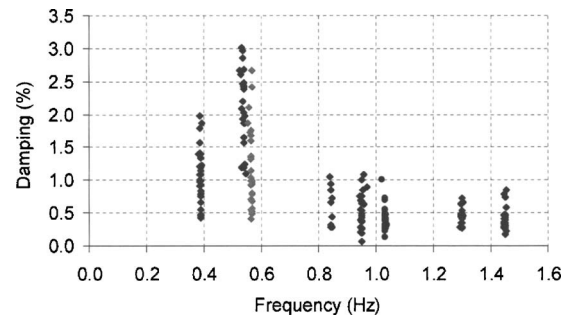


Fig. 10. Dispersion of estimated modal damping coefficients

numerical model in the frequency range of analyses (a model of order 80 has 40 modes). However, the obtained results for $i=10$ were not acceptable. So, the criterion of the maximum order of the estimated models to define the value of i must be adopted with some precaution. It is essential to perform a preliminary analysis to choose the most appropriate value for i .

The data of the 28 setups was processed independently, and therefore 28 stabilization diagrams, like the one presented in Fig. 9, were constructed. This diagram shows that the dynamic behavior of the structure is well represented by state-space models of order between 20 and 40. However, for the setups with higher wind velocities, the vibration of the stay-cables leads to the existence of new poles, making it necessary to use models of higher order (between 40 and 60).

The order of the most suitable model for each setup was selected based on the analysis of the 28 stabilization diagrams, and the natural frequencies and the modal damping ratios of the corresponding models were calculated. Fig. 10 shows the 28 estimates of frequencies and modal damping coefficients for the 8 identified modes. The figure shows that the variation of the natural frequencies along the setups is very small (standard deviations between 0.002 and 0.004 Hz), whereas the modal damping coefficients have a significant dispersion, with standard deviations between 0.11% (mode 7) and 0.60% (mode 3). This dispersion shows how difficult it is to achieve reliable estimates of this parameter, which is justified not only by the uncertainties of the method but also by the variation of damping with the levels of oscillation associated with the wind speed. Columns 7 and 8 of Table 1 present the mean of the identified modal parameters.

The application of the SSI-COV method allowed the identification of two modes with close frequencies (0.548 and 0.568 Hz), but it was not possible to identify the modes with almost coincident frequencies (1.445 Hz and 1.450 Hz), as was also the case with the FDD method. The frequency of 0.845 Hz was identified only in some setups. So, it was possible to obtain estimates for the natural frequency and for the modal damping coefficient. But, it was not possible to have a complete picture of the mode shape.

Data-Driven Stochastic Subspace Identification

The data-driven SSI method (SSI-DATA) is similar to the SSI-COV method. The main difference is that matrices A and C (defined in the previous section) are identified directly using the time series and the concept of projection of subspaces. The algorithm of the method is based on the main theorem of stochastic subspace identification (Overschee and Moor 1996) and involves a QR factorization (the QR factorization expresses any rectangular matrix as the product of an orthogonal or unitary matrix and an upper triangular matrix), a singular value decomposition, and

the resolution of a least-squares equation. After the A and C matrices are identified, the modal parameters are identified as they were in the SSI-COV method.

Several variants of stochastic subspace identification exist. They differ in the weighting of the data matrices (matrices with correlation in the SSI-COV method and matrices with projection in the SSI-DATA). In the present work the SSI-COV was applied using identity weighting matrices, whereas in the SSI-DATA, the weighting matrices associated with the canonical variate analysis (CVA) version of the method (Overschee and Moor 1996) were applied.

The application of this method was also focused on the frequency interval 0–1.6 Hz and even order models of up to 120 were calculated. The best results were achieved with models of orders between 20 and 50 (Magalhães 2004).

As in the application of the SSI-COV method, when the wind velocity increases (after the 13th setup), several new poles appear in the stabilization diagrams to model the vibrations of the cables stimulated by the wind. Inspection of all the stabilization diagrams allowed the identification of the natural frequencies and modal damping ratios presented in columns 9 and 10 of Table 1. The standard deviations of the modal parameters identified by this method are similar to those observed in the application of the SSI-COV method.

The mode shapes estimated by this method show in general a good correlation with those provided by the other applied methods. In particular, the modal assurance criterion (MAC) (Allemang and Brown 1982) between the vertical and torsion mode shapes, estimated by the SSI-DATA and FDD methods, presents values higher than 0.98. The MAC of the 1st transversal bending mode is the lowest and is equal to 0.81. Indeed, observing the mode shapes estimated by all the applied methods, it is concluded that the lateral modes are not as well defined as the vertical or torsion modes, a fact that is probably owing to the lower accelerations registered in the transversal direction.

Identification of the Variation of Modal Damping Ratios with Wind Velocity

In the previous sections the modal damping ratios were identified using the data of the 28 developed setups independently. However, a significant scatter of the identified modal damping coefficients was observed. This dispersion masked the variation of the modal damping coefficients with the wind velocity. In this section, a different approach is adopted to allow the identification of the aerodynamic damping component.

In effect, a considerable number of time series were collected at the reference section during the 28 setups. Using the same precombination technique that was applied before the use of the PP method, it is possible to obtain good estimates of power spectral density functions (PSD) that emphasize the contribution of different types of modes. These spectral density functions can be used to calculate autocorrelation functions, from which modal damping coefficients can be estimated.

For each type of precombined signal, seven PSD functions were estimated, using time series of 4 setups (a total of 84 min). Fig. 11 presents part of the PSD functions of the half sum of vertical acceleration, as well as the mean wind velocity associated with each spectrum. In this figure, it is interesting to observe that the spectra associated with lower wind velocities are characterized by sharper peaks at the resonant frequencies.

Fig. 12 shows the variation of the modal damping coefficients of the three analyzed modes with the wind velocity, using the

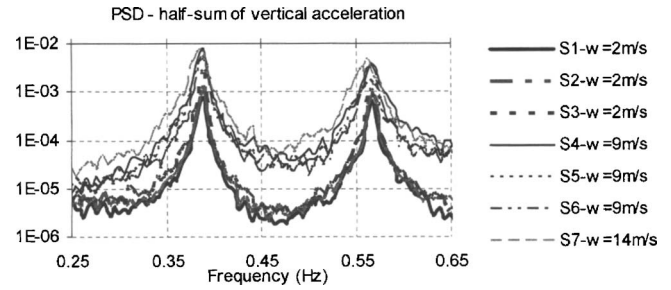


Fig. 11. Power spectral density functions of half-sum of vertical acceleration at reference section

mean values of the ratios calculated from spectra associated with equal mean wind velocities. It is clear that the damping of the vertical and transversal modes increases with the wind velocity, while the damping of the torsion mode is almost constant. These results are coherent with the quasi-steady theory of aerodynamic damping (Simiu and Scanlan 1996), and they will be investigated more deeply after the installation of the long-term monitoring system of the bridge, presently under development (Caetano et al. 2005).

Conclusions

This paper presents the analysis of data collected in the ambient vibration test of the International Guadiana cable-stayed bridge using some of the most powerful output-only identification techniques: peak-picking (PP), frequency domain decomposition (FDD), covariance-driven stochastic subspace identification (SSI-COV), and data-driven stochastic subspace identification (SSI-DATA). The most important results are summarized in Table 1 and lead to the following major conclusions:

- The peak-picking method, together with a suitable precombination of the collected signals, allows the identification of modes with close frequencies. However, this method is efficient only in structures where the different types of modes are independent and where the closely spaced modes are not of the same type (vertical bending, lateral bending, or torsion). In these cases more developed stochastic identification techniques have to be applied to ensure a correct identification of all modes.
- In structures where cable vibrations occur, the standard procedure of PP and FDD methods to identify the natural frequencies (using the peaks of the ANPSD function or the peaks of the average normalized singular values of the spectrum matrix)

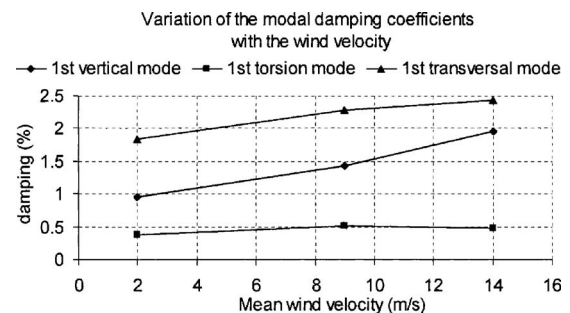


Fig. 12. Variation of modal damping coefficients with wind velocity

ces) is not sufficient. A more detailed analysis is needed to successfully separate the local cable frequencies from the global bridge frequencies. In particular, it is important to analyze spectra associated with different wind conditions. In this context, the use of color maps with the evolution of the spectra along the performed setups is suggestive.

- The theory behind the FDD, SSI-COV, and SSI-DATA methods enables the identification of closely spaced modes. In the case of the first pair of closely spaced modes of the tested bridge ($f=0.537$ Hz and $f=0.566$ Hz) good results have been obtained. However, worse results were obtained for the second pair of modes (second transversal mode and first torsion mode), with identification of a mode that is a combination of the configurations of the two modes characterized by the numerical model. This happened because the natural frequencies of these modes are almost coincident and so they became coupled, as they are simultaneously stimulated by the ambient excitation.
- The vibrations of stay-cables induced by moderate wind velocities lead to the existence of several poles of the state-space models that have a negative effect on the performance of the SSI methods, forcing the use of models of very high order. Consequently, this significantly increases the calculation effort and the difficulty in the selection of poles with global structural meaning. Alternatively, it forces concentration of the analysis on a narrow frequency interval.
- In general a good agreement exists between the identified modal properties obtained by the different techniques. The modal damping ratios are the identified parameters that exhibit the most significant scatter.
- An appropriate analysis of the data from ambient vibration testing allows the evaluation of the variation of modal damping ratios with the wind velocity and estimation of the aerodynamic damping component. However, it is important to use, for that purpose, longer time series than those used to identify natural frequencies and mode shapes. Nevertheless, the level of scatter of damping estimates shows that the identification of a modal damping coefficient based on ambient vibration testing needs further research before damping identification tools as reliable as those provided by free vibration or harmonic excitation tests can be created.
- All the referred identification techniques require human judgment in several steps of the application of the algorithms. For instance, in the peak-picking method the peaks associated with global modes have to be chosen, in the FDD method the singular values used to estimate the modal correlation functions have to be defined, in the SSI method the order of the models used in each setup has to be selected. Thus, the results of each method are dependent on the experience of the user. Therefore, it is important to improve the existing techniques to enable the automatic application of output-only modal identification methods and to integrate them in implemented continuous monitoring programs.

Acknowledgments

This work was developed in the context of the Research Project (POCTI/ECM/46475/2002) "Vibrations in Cable-Stayed Bridges" and of the Ph.D. scholarship provided to the first author (SFRH/BD/24423/2005). The writers acknowledge the corresponding funding support provided by the Portuguese Foundation for Science and Technology (FCT).

References

- Abdel-Ghaffar, A. M., and Housner, G. W. (1978). "Ambient vibration tests of suspension bridges." *J. Engrg. Mech. Div.*, 104, 983–999.
- Allemang, R. J., and Brown, D. L. (1982). *A correlation coefficient for modal vector analysis*, IMAC I, Orlando, Fla.
- Andersen, P. (1997). "Identification of civil engineering structures using vector ARMA models." Ph.D. thesis, Aalborg Univ., Aalborg, Denmark.
- Asmussen, J. C. (1997). "Modal analysis based on the random decrement technique." Ph.D. thesis, Aalborg Univ., Aalborg, Denmark.
- Bendat, J., and Piersol, A. (1971). *Random data: Analysis and measurements procedures*, Wiley.
- Bendat, J., and Piersol, A. (1980). *Engineering applications of correlation and spectral analysis*, Wiley.
- Branden, B., Peeters, B., and Roeck, G. De. (1999). *Introduction to Macec, Technical Rep.*, Katholieke Univ. Leuven.
- Brincker, R., Ventura, C., and Andersen, P. (2001). "Damping estimation by frequency domain decomposition." *Proc., IMAC XIX*, Kissimmee, Fla.
- Brincker, R., Zhang, L., and Andersen, P. (2000). "Modal identification from ambient responses using frequency domain decomposition." *Proc., IMAC XVIII*, San Antonio, Tex.
- Caetano, E., and Cunha, A. (2003). "Identification of Parametric Excitation at the Int. Guadiana Bridge." *Proc., Int. Conf. on Cable Dynamics*, Santa Margherita, Italy.
- Caetano, E., Cunha, A., and Magalhães, F. (2005). *Vibration assessment of the Int. Guadiana Bridge*, Eurodyn 2005, Paris, France.
- Corrêa, M. R., and Costa, A. C. (1992). "Dynamic tests of the bridge over the Arade river." *Cable-stayed bridges of Guadiana and Arade* (in Portuguese).
- Crawford, R., and Ward, H. S. (1964). "Determination of the natural period of buildings." *Bull. Seismol. Soc. Am.*, 54(6), 1743–1756.
- Cunha, A., Caetano, E., and Delgado, R. (2001). "Dynamic tests on a large cable-stayed bridge." *J. Bridge Eng.*, 6(1), 54–62.
- Felber, A. J. (1993). "Development of a hybrid bridge evaluation system." Ph.D. thesis, Univ. of British Columbia, Vancouver, Canada.
- Felber, A. J., and Cantieni, R. (1996). *Introduction of a New Ambient Vibration Testing System: Description of the System and Seven Bridges Tests, Technical Rep. 156'521*, EMPA, Dübendorf, Switzerland.
- LMS Int. (2005). LMS Test Lab, Leuven, Belgium.
- McLamore, V. R., Hart, G. C., and Stubbs, I. R. (1971). "Ambient vibration of two suspension bridges." *J. Struct. Div.*, 97(10), 2567–2582.
- Magalhães, F. (2004). "Stochastic modal identification for the validation of numerical models." MS thesis, Univ. of Porto, Porto (in Portuguese).
- Overschee, P. V., and Moor, B. De. (1996). *Subspace identification for linear systems*, Kluwer Academic Publishers, Leuven, Belgium.
- Peeters, B. (2000). "System identification and damage detection in civil engineering." Ph.D. thesis, Katholieke Univ. Leuven, Leuven.
- Peeters, B., and Auweraer, H. Van der. (2005). "PolyMAX: A Revolution in Operational Modal Analysis." *Proc., 1st Int. Operational Modal Analysis Conf. (IOMAC)*, Copenhagen, Denmark.
- Piombo, B., Giorcelli, E., Garibaldi, L., and Fasana, A. (1993). *Structures identification using ARMAV models*, IMAC XI, Kissimmee, Fla.
- Prevosto, M. (1982). "Algorithmes d'Identification des Caractéristiques Vibratoires de Structures Mécaniques Complexes." Ph.D. thesis, Univ. de Rennes I, France.
- Simiu, E., and Scanlan, R. (1996). *Winds effects on structures: Fundamentals and applications to design*, Wiley.
- Structural Vibration Solutions (SVS). (1999–2004). *ARTeMIS Extractor Pro, Release 3.41*, Aalborg, Denmark.
- Trifunac, M. (1972). "Comparison between ambient and forced vibration experiments." *Earthquake Eng. Struct. Dyn.*, 1, 133–150.
- Welch, P. D. (1967). "The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short modified periodograms." *IEEE Trans. Audio Electroacoust.*, AU-15(2).