Crystal Clear SSI for Operational Modal Analysis of Aerospace Vehicles*

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Abstract

In this paper we revisit the problem of the modal analysis of space launchers. We consider the Ariane 5 launcher with its usual equipment during a commercial flight under the natural unknown excitation. The case of space launchers is a typical example of a complex structure with sub-structures strongly and quickly varying in time. This issue becomes especially important in e.g. estimation of damping of aerospace vehicles. The eigenfrequencies are also sliding during the flight but the modeshapes are more stable. Recently, a new implementation of the subspace identification method has been proposed, leading to cleaner and more stable stabilization diagrams. We monitor the behavior of estimated modal parameters by applying this "crystal clear" implementation of the data driven and the covariance driven Stochastic Subspace Identification algorithms. We show the importance of "crystal clear" to monitor successfully frequencies and damping estimates over time in such a non stationary case.

1 INTRODUCTION

In modal analysis of vibrating structures it is usual that operating conditions differ completely from those of laboratory experiments. The first major difference is that under natural loading conditions, excitations cannot be measured and are usually non-stationary; this does not mean that laboratory results are not valid but that in-operation treatment needs different techniques. Subspace-based algorithms are currently used and have been proven efficient for modal parameter estimation (natural frequencies, damping ratios, modeshapes). In this paper, we use output-only covariance-driven and data-driven Stochastic Subspace Identification (SSI) methods for the identification of the modal parameters.

These methods assume that the structure is stationary. Even for many non-stationary structures, e.g. aircraft, the stationary assumption can be still assumed because the non-stationarity is due to the load which varies slowly. However, in our case the structure is not stationary. The mass of the launcher is strongly and quickly varying in time. Nevertheless, we want to apply SSI methods to identify the modal parameters.

In the case of Ariane we could have some positive points for the method. The output are continuously measured and so it is possible to use sliding windows and to follow the time evolution of a specific eigenfrequency. Moreover modeshapes are more slowly varying than the eigenfrequencies and it is easier (if possible) to compare and follow the modeshapes. Another important point is that Ariane is a complex structure with different sub-structures and the sub-structures are not simultaneously varying: for example, during the initial part of the flight, the boosters are burning and strongly varying but the other parts of the launcher can be considered as stationary parts excited by the boosters.

A preceding analysis of the data of Ariane 5 was made in [8] and [9]. The current paper extends the previous research by the analysis with the data-driven SSI algorithm, which is recalled briefly in the first part of this paper together with the covariance-driven SSI procedure. We present some results of the modal parameter identification for Ariane 5 using also the recently developed "crystal clear" implementation [2] which is especially helpful in the presence of non-stationary data and it leads to cleaner and more stable stabilization diagrams.

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2 IDENTIFICATION PROCEDURE

2.1 Modeling

The mechanical system is supposed to be a stationary linear dynamical system

$$\begin{cases} M\ddot{\mathcal{Z}}(t) + C\dot{\mathcal{Z}}(t) + K\mathcal{Z}(t) &= \nu(t) \\ Y(t) &= L\mathcal{Z}(t) \end{cases},$$

with

- Z: displacements of the degrees of freedom,
- M, C, K: mass, damping, stiffness matrices,
- t: continuous time,
- ν: excitation,
- L: observation matrix giving the observation Y.

The modal characteristics

- μ vibration modes or eigenfrequencies
- ψ_μ modal shapes or eigenvectors

are solutions of the following equation:

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \ \psi_\mu = L\Psi_\mu.$$

We switch to the state space model in discrete time by sampling at the rate $1/\delta$ with

$$X_k = \begin{bmatrix} \mathcal{Z}(k\delta) \\ \dot{\mathcal{Z}}(k\delta) \end{bmatrix} \ , \ Y_k = Y(k\delta)$$

and get

$$\begin{cases} X_{k+1} &= FX_k + V_k \\ Y_k &= HX_k \end{cases}$$

with

$$F = \exp(A\delta) \quad \text{and} \quad H = \begin{bmatrix} L & 0 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

The input noise is assumed to have zero-mean and its covariance is:

$$\begin{aligned} Q_V(k) = & \int_{k\delta}^{(k+1)\delta} \exp(As)\tilde{Q}(s)\exp(A^T s)ds \\ \tilde{Q}(s) = & \begin{bmatrix} 0 & 0 \\ 0 & M^{-1}Q_\nu(s)M^{-T} \end{bmatrix} \end{aligned}$$

$Q_\nu(s)$ is the covariance matrix of ν. The modal characteristics (μ, ψ_μ) are given by the eigenstructure (λ, Φ_λ) of F:

$$\begin{aligned} e^{\delta\mu} &= \lambda \\ \psi_\mu &= \phi_\lambda \stackrel{\Delta}{=} H\Phi_\lambda \end{aligned}$$

In the sequel the dimension of the observed output Y is much smaller than the dimension of the state X.

2.2 Output-only subspace-based modal analysis

We consider the previously defined discrete time model in state space form:

$$\left\{ \begin{array}{rcl} X_{k+1} & = & F\,X_k + V_{k+1} \\ Y_k & = & H\,X_k \end{array} \right. \tag{1}$$

Only knowing the output data Y_k at the time instants $k = 1, \ldots, N$, we want to identify the eigenstructure (λ, ϕ_λ) of the system with Stochastic Subspace Identification algorithms. For doing so, we follow two approaches: the covariance driven [4, 1] and data driven approach with the Unweighted Principal Component algorithm [10].

For both approaches we choose p and q as variables with $p+1 \geq q$ that indicate the quality of the estimations (a bigger p leads to better estimates) and the maximal system order ($\leq qr$ with the number of sensors r). Normally, we choose $p = q-1$, but in the case of measurement noise $p = q-1+l$ should be chosen, where l is the order of the noise. We build the data matrices

$$\mathcal{Y}^+_{p+1} \stackrel{\text{def}}{=} \begin{pmatrix} Y_{q+1} & Y_{q+2} & \vdots & Y_{N-p} \\ Y_{q+2} & Y_{q+3} & \vdots & Y_{N-p+1} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{q+p+1} & Y_{q+p+2} & \vdots & Y_N \end{pmatrix}, \quad \text{and} \quad \mathcal{Y}^-_{q} \stackrel{\text{def}}{=} \begin{pmatrix} Y_{q} & Y_{q+1} & \vdots & Y_{N-p-1} \\ Y_{q-1} & Y_{q} & \vdots & Y_{N-p-2} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{1} & Y_{2} & \vdots & Y_{N-p-q} \end{pmatrix} \tag{2}$$

and, according to the method, a Hankel or a weighted Hankel matrix as follows:

- For the **covariance driven** approach, we build the Hankel matrix

$$\mathcal{H}^{\text{cov}}_{p+1,q} \stackrel{\text{def}}{=} \mathcal{Y}^+_{p+1} \mathcal{Y}^{-\,T}_{q} = \begin{pmatrix} R_1 & R_2 & R_3 & \ldots & R_q \\ R_2 & R_3 & R_4 & \ldots & \vdots \\ R_3 & R_4 & R_5 & \ldots & \vdots \\ \vdots & \vdots & \ddots & \ldots & \vdots \\ R_{p+1} & \vdots & \ddots & \ldots & R_{p+q} \end{pmatrix},$$

  where $R_i = \mathbf{E}\,\left(Y_k\,Y^T_{k-i}\right)$ is the correlation of the output data and $\mathbf{E}$ is the expectation operator. The matrix $\mathcal{H}^{\text{cov}}_{p+1,q}$ has the factorization property

$$\mathcal{H}^{\text{cov}}_{p+1,q} = \mathcal{O}_{p+1}\,F\,\mathcal{C}_q$$

  with the matrix of observability

$$\mathcal{O}_{p+1} = \begin{pmatrix} H \\ HF \\ \vdots \\ HF^p \end{pmatrix}$$

  and the matrix of controllability $\mathcal{C}_q$.

- For the Unweighted Principle Component algorithm of the **data driven** approach, we build the weighted Hankel matrix[1]

$$\mathcal{H}^{\text{data}}_{p+1,q} \stackrel{\text{def}}{=} \mathcal{Y}^+_{p+1} \mathcal{Y}^{-\,T}_{q} \left(\mathcal{Y}^-_{q} \mathcal{Y}^{-\,T}_{q}\right)^{-1} \mathcal{Y}^-_{q}$$

  The matrix $\mathcal{H}^{\text{data}}_{p+1,q}$ enjoys the factorization property

$$\mathcal{H}^{\text{data}}_{p+1,q} = \mathcal{O}_{p+1} \mathcal{X}_q$$

  into matrix of observability and Kalman filter state sequence.

[1] As $\mathcal{H}^{\text{data}}_{p+1,q}$ is usually a very big matrix and difficult to handle, we continue the calculation in practice with the R part from an RQ-decomposition of the data matrices, see [10] for details. This will lead to the same results as for the system identification only the left part of the decomposition of $\mathcal{H}_{p+1,q}$ is needed.

In what follows, we skip the superscripts and subscripts of the Hankel or weighted Hankel matrix $\mathcal{H}$, as the following identification procedure is the same for the covariance and data driven approach. Now we want to obtain the eigenstructure of the system (1) from a given matrix $\mathcal{H}$. The observability matrix $\mathcal{O}_{p+1}$ is obtained from a thin SVD of the matrix $\mathcal{H}$ and its truncation at the desired model order:

$$\begin{aligned} \mathcal{H} &= U\,\Delta\,V^T \\ &= (U_1 \quad U_0) \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} V^T, \\ \mathcal{O}_{p+1} &= U_1\,\Delta_1^{1/2}. \end{aligned} \tag{3}$$

The observation matrix H is then found in the first block-row of the observability matrix $\mathcal{O}_{p+1}$. The state-transition matrix F is obtained from the shift invariance property of $\mathcal{O}_{p+1}$, namely

$$\mathcal{O}_p^{\uparrow}(H,F) = \mathcal{O}_p(H,F)\;F, \quad \text{where} \;\; \mathcal{O}_p^{\uparrow}(H,F) \stackrel{\text{def}}{=} \begin{pmatrix} HF \\ HF^2 \\ \vdots \\ HF^p \end{pmatrix}. \tag{4}$$

Of course, for recovering F, it is needed to assume that $\operatorname{rank}(\mathcal{O}_p) = \dim F$, and thus that the number $p+1$ of block-rows in $\mathcal{H}$ is large enough. The eigenstructure (λ, ϕ_λ) results from

$$\det(F - \lambda\,I) = 0, \;\; F\,\varphi_\lambda = \lambda\,\varphi_\lambda, \;\; \phi_\lambda = H\varphi_\lambda, \tag{5}$$

where λ ranges over the set of eigenvalues of F.

In practice, we increase the truncation order of the SVD from 1 to the maximal system order in Equation (3) and get a stabilization diagram of the obtained modes vs model order. This gives results for successive different but redundant models and we can distinguish the modes that are common to many successive models from the spurious modes. This step gives frequency bands corresponding to the identified natural frequencies. Then we can select such a frequency band and plot the MACs for the modeshapes corresponding to the frequencies of the band. This information indicates whether the modes for all the orders agree on the same mode shape and are hence part of the modal signature.

There are many papers on the used identification techniques. A complete description can be found in [4], [5], [6], [7], [10] and the related references. A proof of non-stationary consistency of these subspace methods can be found in [11].

3 THE EXPERIMENTAL CASE: ARIANE 5

3.1 Ariane 5

Ariane 5 is a launch vehicle under ESA's responsibility with CNES as prime contractor. Aerospatiale is the industrial architect for the complete vehicle and prime contractor for parts of the launcher.

Ariane 5's lower section consists of a cryogenic central main (EPC) stage fueled by liquid hydrogen and liquid oxygen, plus two solid boosters(EAP). There is an upper stage using storable propellants. The vehicle is also fitted with a bearing structure (Speltra), a fairing and an equipment bay.

The structure is equipped with more than 100 sensors. The measurements are of different types: acceleration, constraint, displacement or mechanical vibration. The number of sensors decreases during the flight as the used parts separate from the launcher. For the same reason the length of the records depends on the location of the measurement on the vehicle.

The difference with ground modal analysis (such as an aircraft certification) is that the locations of sensors are imposed and chosen for other purposes than modal analysis (mainly for control). Due to mechanical constraints some parts of the vehicle cannot be equipped. Moreover 100 is a low number of sensors for such a complex structure. There are about 30 "interesting" modes for the structure and with only 100 sensors they cannot be all distinguished. Moreover, each mode is observed only by few sensors as many modes are purely local. Hence it is only possible to examine MAC correlations of identified modes when some sensors (e.g. 4) are used for the identification of the modes.

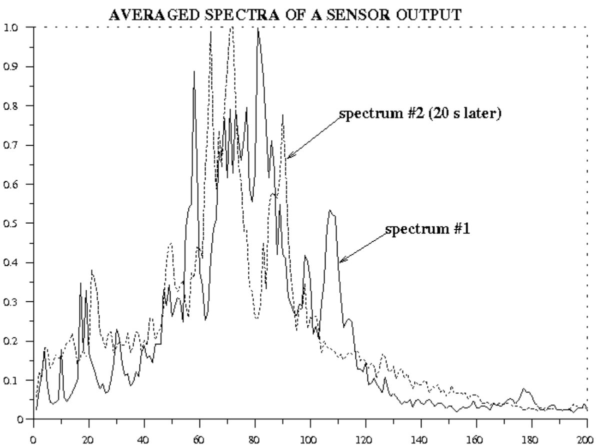

Figure 1: Effect of frequency sliding on successive spectra

3.2 Signal examination and preprocessing

The output of 103 sensors were available for this study. The goal of the results presented here is not the full modal analysis of Ariane 5 but only to demonstrate the capabilities of the methods. We will use only 6 sensors chosen for some important modes considered difficult to be identified.

There are 3 different periods for the flight: the starting phase, the flight with the boosters (EAP flight) and the long phase with the main cryogenic stage (EPC flight).

Figure 1 shows the sliding behavior of frequencies during the flight; on the same display we compute 2 averaged spectra. Each spectrum is computed for a block of 20s and the second is computed 20s after the first one. It is clear that the variation of frequencies depends on the modes and on the period of the flight.

4 RESULTS AND DISCUSSION

4.1 Identification parameters

- Choose the records used for the identification: the number and location of sensors. The best results are obtained with a low number of sensors: 2 to 10; generally 2, 3 or 4 are good choices. The choice of the sensor locations is important as well and a specialist was requested to do the best choice corresponding to the modal parameters we want to identify due to the existence of local modes. This is a crucial point for Ariane and the choices were done by the experts of Aerospatiale and CNES. In most cases, the best results for natural frequencies and damping ratios are obtained with 2 sensors.
- Perform the system identification procedure of Section 2.2. The Hankel and weighted Hankel matrices for the covariance-driven and data-driven approach are built and the stabilization diagrams containing the modal parameters computed.
- Signature selection: it is automatically done after the choice of a window size and the number of occurrences in the stabilization diagram. We find the natural frequencies located in a frequency window and compare the number of these occurrences to the number of model sizes.

  The best results are obtained with the “crystal clear” selection where an approximate mean square solution is considered when the system transition matrix is computed (presented in [2]).
- Local examination: for every frequency previously selected we plot the result of the identification w.r.t. the state-space order, i.e. the natural frequency, the damping coefficient or the MAC value.

4.2 Some results

All the stabilization diagrams in this section are obtained with the modal analysis toolbox COSMAD [12], [13] which is freely available at www.irisa.fr/i4s/cosmad. For the data-driven approach, we also include some results obtained with the commercial software Artemis (www.svibs.com).

4.2.1 Covariance driven method

The effect of low-pass-filtering the data and projection stabilization (crystal clear, CC algorithm) are illustrated by Figures 2, 3, 4 and 5.

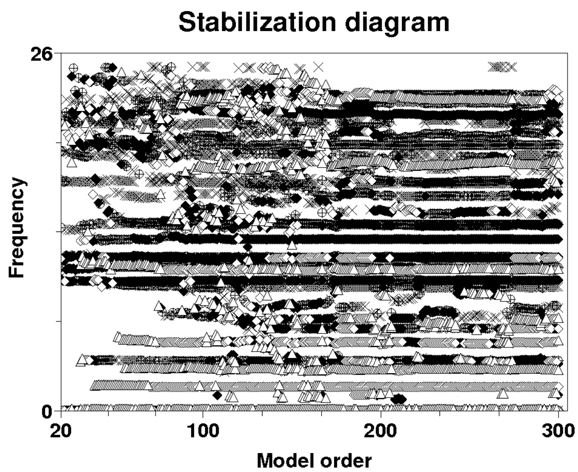

Figure 2: Stabilization diagram - unfiltered data

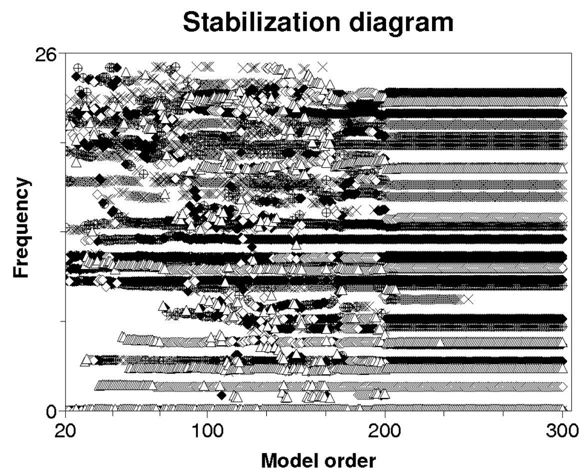

Figure 3: Stabilization diagram - unfiltered data - CC algorithm

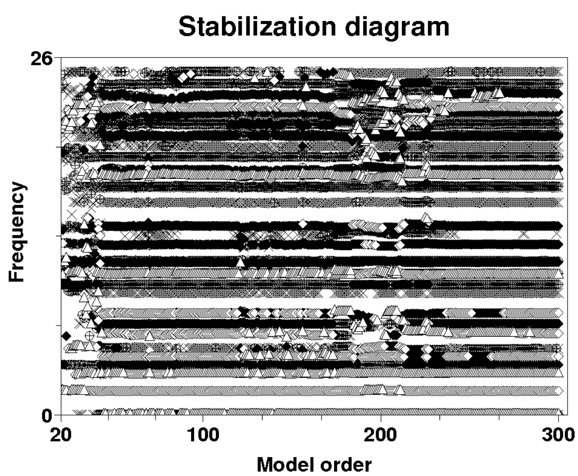

Figure 4: Stabilization diagram - filtered data

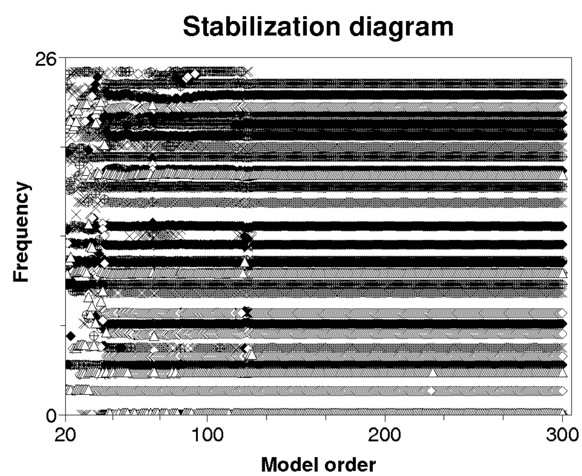

Figure 5: Stabilization diagram - filtered data - CC algorithm

The alignment for the lowest frequencies and the corresponding damping for the first mode are shown on Figures 6 and 7.

On Figures 8 and 9 the identification at two different periods shows the evolution of some modes, while some other remain constant. We can also follow the evolution of a mode during the flight period using an automatic monitoring procedure that shifts a window over the data processed by the identification algorithm. Here the CC algorithm was essential for obtaining good results, otherwise the modal parameters did not stabilize sufficiently for the automatic monitoring procedure. The results for the first mode are shown in Figures 10 and 11. Both the natural frequency and the damping increase, while the fluctuation of the damping estimates is much higher than of the frequency estimates.

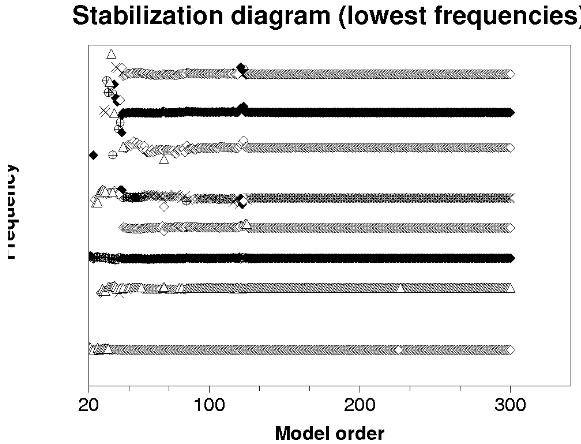

Figure 6: Lowest eigenfrequencies - CC algorithm

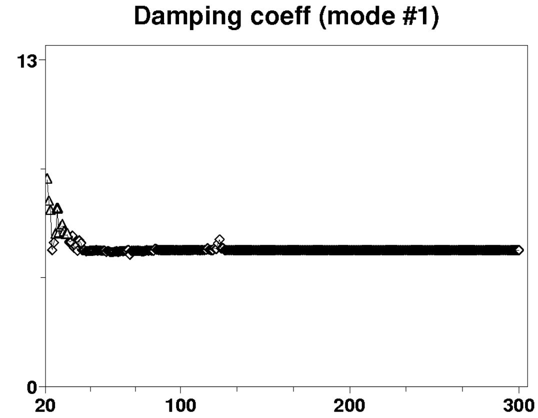

Figure 7: Damping of first mode - CC algorithm

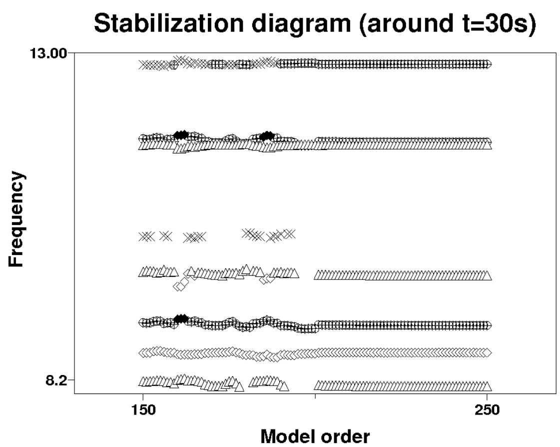

Figure 8: Eigenfrequencies at t=30s - CC algorithm

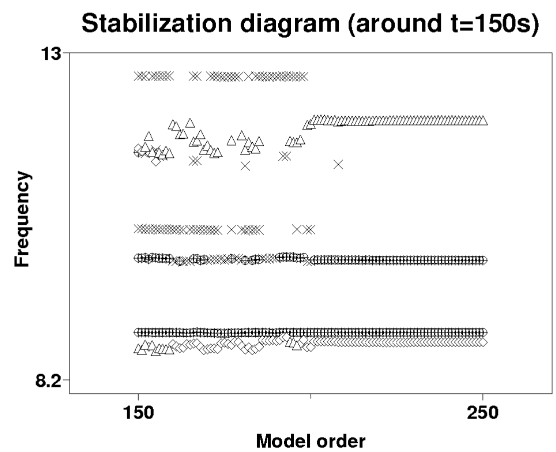

Figure 9: Eigenfrequencies at t=150s - CC algorithm

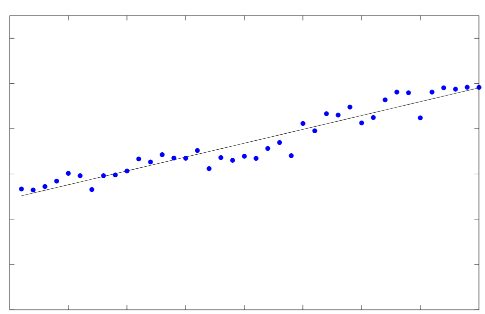

Figure 10: Mode #1 Monitoring with CC algorithm - Evolution of the frequency

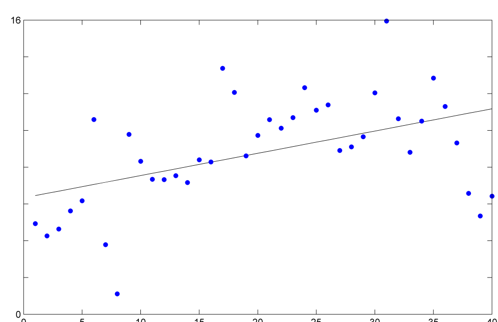

Figure 11: Mode #1 Monitoring with CC algorithm - Evolution of the damping

4.2.2 Data driven method

We compare the covariance and data driven methods applied to the same data and using the same tuning for the parameters. The respective stabilization diagrams are on Figure 5 for the covariance driven approach and Figure 12 for the data driven approach. The corresponding damping evolutions for the first mode are represented on Figures 7 and 13.

We also present some results of the modal parameter estimation of the Ariane data processed with the crystal clear UPC algorithm with the commercial software Artemis on Figures 14, 15 and 16. Note that in the first two figures the frequency is plotted on the horizontal axis and the model order on the vertical axis.

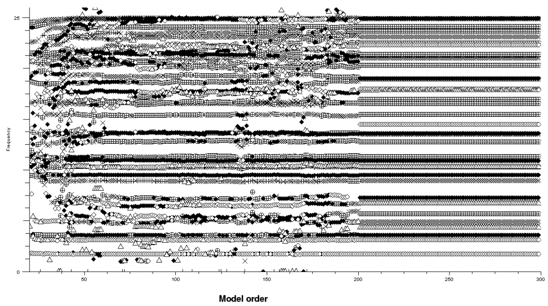

Figure 12: Data-driven - Filtered data - CC algorithm

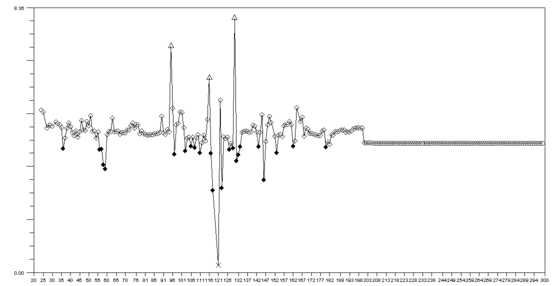

Figure 13: Data-driven - Damping of first mode - CC algorithm

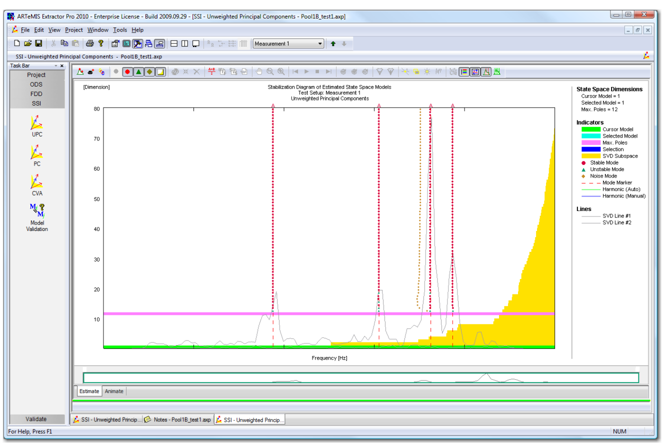

Figure 14: Stabilization diagram for the low modes from Artemis

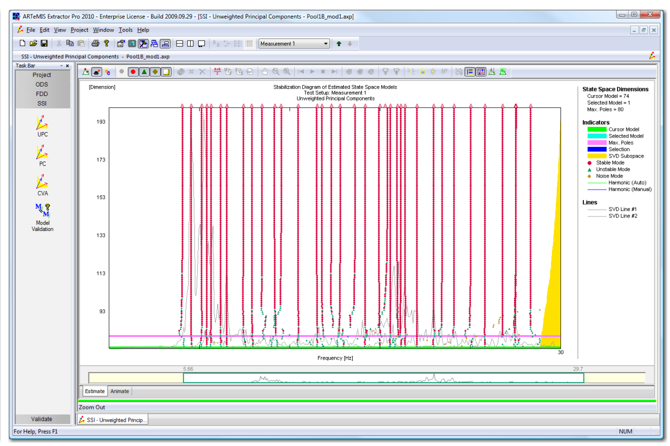

Figure 15: Stabilization diagram for the high modes from Artemis

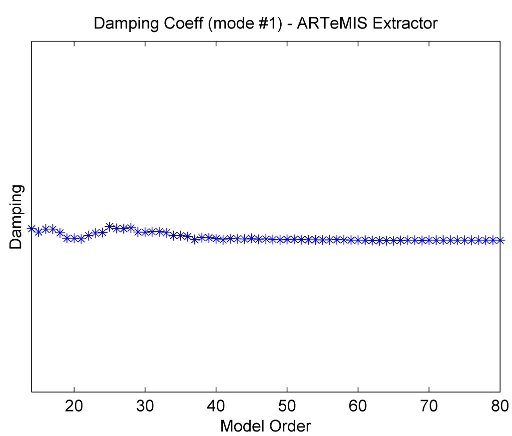

Figure 16: Damping of first mode (Artemis)

4.2.3 Comments

Both the covariance- and the data-driven methods gave similar results for the stabilization diagrams, especially for the estimation of the frequencies. Damping estimates benefit a lot from the "crystal clear" implementation in both methods, even with this very nonstationary case study. In general, the crystal clear procedure (CC) is a major improvement with respect to the basic subspace identification algorithm. It helped to get clear results even in the case of nonstationary data, and a monitoring of the structure during intense changes of the modal parameters as in Figures 10 and 11 was still possible due to the CC procedure. It may be mandatory for an automated monitoring procedure.

5 Conclusions

We have presented some capabilities of automated subspace identification methods for non stationary structures. We succeeded in the identification of a quickly varying structure and could follow the evolution of the system in continuous time with the help of the "crystal clear" algorithm. Both the covariance- and data-driven approach gave satisfying results, while the covariance-driven approach turned out to give more stable damping estimations.

With the help of specialists in the processing procedure, the results for Ariane 5 are good. The main problem is not the variation of the structure but the location of sensors. These locations have been chosen for other purposes than modal analysis and are hard constraints.

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